School on Disordered Media
Short Talks
Erdős Center
Budapest, Hungary, 20 January - 24 January 2025

Monday

Grega Saksida (University of Warwick)
Lace expansion in statistical mechanics

Abstract: Lace expansion is a powerful technique first developed by David Brydges and Thomas Spencer in 1985 to study self-avoiding random walks. In 2007, Akira Sakai found a way to apply it to the classical Ising model. The technique has been used on other models as well since then. I will give a short demonstration of the technique on the self- avoiding walk, and show how we hope to use the technique next.

Léonie Papon (Durham University)
Interface scaling limit for the critical planar Ising model perturbed by a magnetic field

Abstract: In this talk, I will consider the interface separating +1 and -1 spins in the critical planar Ising model with Dobrushin boundary conditions perturbed by an external magnetic field. I will prove that this interface has a scaling limit. This result holds when the Ising model is defined on a bounded and simply connected subgraph of $\delta\mathbb{Z}^2$, with $\delta > 0$. I will show that if the scaling of the external field is of order $\delta^{15/8}$, then, as $\delta \to 0$, the interface converges in law to a random curve whose law is conformally covariant and absolutely continuous with respect to SLE_3. This limiting law is a massive version of SLE_3 in the sense of Makarov and Smirnov and I will give an explicit expression for its Radon-Nikodym derivative with respect to SLE_3. I will also prove that if the scaling of the external field is of order $\delta^{15/8}g(\delta)$ with $g(\delta) \to 0$, then the interface converges in law to SLE_3. In contrast, I will show that if the scaling of the external field is of order $\delta^{15/8}f(\delta)$ with $f(\delta) \to \infty$, then the interface degenerates to a boundary arc.

Kieran Ryan (TU Vienna)
Conformally invariant boundary arcs in double dimer models

Abstract: We study two different double dimer models which, along with the usual non-crossing loops and doubled edges, exhibit arcs which begin and end at a certain portion of the boundary. We argue that these models satisfy a discrete version of a coupling between the Arc Loop Ensemble (ALE) and two Gaussian Free Fields with different boundary conditions. We prove that certain statistics of the arcs are conformally invariant in the small mesh limit, giving the same limit from both discrete

models, and equal to the corresponding ALE statistics. Joint work with Marcin Lis and Lucas Rey

Zsuzsanna Baran (University of Cambridge)
Phase transition for random walks on graphs with an added weighted random matching

Abstract: For a finite graph $G = (V, E)$, we let G^* be the random graph obtained by placing edges of weight ε between pairs of vertices of a random perfect matching. For two families of graphs we establish a phase transition in the occurrence of cutoff of a weighted random walk on G^* in terms of the weight ε. We also give a general condition on ε that is sufficient to ensure cutoff. Joint work with Jonathan Hermon, Andela Sarkovic, and Perla Sousi.

Georgios Athanasopoulos (Roma Tre University)
The exact solution of the 2D classical Ising model via the Kac-Ward method

Abstract: Onsager proposed a closed-form expression for the free energy of the 2D classical Ising model in 1944. In 1952, Kac and Ward introduced an alternative elegant method of combinatorial nature. Kager, Lis and Meester in 2013 and Aizenman and Warzel in 2018 gave a rigorous proof of the latter. We extend their result to the triangular lattice as well as with coupling constants of arbitary sign.

Kesav Krishnan (University of Victoria)
Ground State of the Disordered Monomer Model

Abstract: We prove that the disordered monomer-dimer model does not admit infinite volume incongruent ground states in $\mathbb{Z}^d$ which can be obtained as a limit of finite volume ground states. Furthermore, we also prove that these ground states are stable under perturbation of the weights in a precise sense. As an application, we obtain a CLT for the ground state weight for a growing sequence of tori. Our motivation stems from a similar and long standing open question for the short range Edwards-Anderson spin glass model.

Kevin Hu (Brown University)
An H-theorem for a conditional McKean-Vlasov process related to interacting diffusions on regular trees

Abstract: We study the long-time behavior of the κ-Markov local-field equation (κ-MLFE), which is a conditional McKean-Vlasov equation associated with interacting diffusions on the κ-regular tree. Under suitable assumptions on the coefficients, we prove well-posedness of the κ-MLFE. We also establish an H-theorem by identifying an energy functional, referred to as the sparse free energy, whose derivative along the measure flow of the κ-MLFE is given by a nonnegative functional that can be viewed as a modified Fisher information. Moreover, we show that the zeros of the

latter functional coincide with the set of stationary distributions of the κ-MLFE and are also marginals of splitting Gibbs measures on the κ-regular tree. Furthermore, we show that for a natural class of initial conditions, the corresponding measure flow converges to one of the stationary distributions, thus demonstrating that the sparse free energy acts as a global Lyapunov function. Under mild additional conditions, in the case $\kappa = 2$ we prove that the sparse free energy arises naturally as the renormalized limit of certain relative entropies. We exploit this characterization to prove a modified logarithmic Sobolev inequality and establish an exponential rate of convergence of the 2-MLFE measure flow to its unique stationary distribution.

Vilas Winstein (University of California, Berkeley)
Independent sets in a percolated hypercube

Abstract: Independent sets have long been studied as a model for gases in statistical mechanics, and Sapozhenko's graph container method was developed to understand their quantity in the hypercube, where they correspond to error-detecting codes. By removing edges at random (i.e. percolating the hypercube), this quantity becomes the Gibbs measure of a disordered system, analogous to models of recent interest such as spin glasses. This was first explored by Kronenberg and Spinka, who used tools from statistical mechanics to calculate the moments of the (random) number of independent sets in a percolated hypercube and observed interesting phase transitions in the concentration of this quantity and in the appearance of various local structures in a typical independent set.

Aided by their results, we build a probabilistic framework to establish sharp distributional characterizations of the number of independent sets, as well as structure theorems about the geometry of a typical one. This settles several open problems and explains how the energy landscape changes with the percolation parameter, thereby leading to the phase transitions observed in prior work.

Tuesday

Luca Makowiec (National University of Singapore)
Diameter of Random Spanning Trees in Random Environment

Abstract: We will introduce Random Spanning Trees in Random Environment, a disordered system on spanning trees. Our primary goal is to determine the order of the diameter of a (typical) spanning tree, a crucial step towards the pursuit of a non-trivial scaling limit. For the complete graph, we will give upper and lower bounds for a phase transition where we either observe the diameter of the Uniform Spanning Tree or that of the Minimum Spanning Tree, which scale with different power laws. Lastly, we discuss a conjecture about the order of the diameter inside the critical window.

Ágnes Kúsz (Budapest University of Technology and Economics)
Random spanning trees interpolating between the UST and the MST of the complete graph

Abstract: We introduce $\mathrm{WST}^{\beta_n}(K_n)$ as the weighted spanning tree of the complete graph K_n w.r.t. the random electric network of conductances $\{\exp(-\beta_n U_e)\}_{e\in E(K_n)}$ with Unif$[0,1]$ i.i.d. U_e's.

Moving from $\beta_n \equiv 0$ to faster and faster growing β_n's, the model interpolates between the *uniform* and the *minimum* spanning trees: $\mathrm{WST}^0(K_n) = \mathrm{UST}(K_n)$, and there are phase transitions for $\mathrm{WST}^{\beta_n}(K_n)$ behaving more and more like $\mathrm{MST}(K_n)$

> around $\beta_n = n^{3+o(1)}$ regarding the agreement of the two standard algorithms generating these models : Aldous-Broder and Prim's invasion algorithms,
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> around $\beta_n = n^{2+o(1)}$ regarding the models consisting of exactly the same edges, and
>
> around $\beta_n = n^{1+o(1)}$ regarding the expected total length $\mathbb{E}\left[\sum_{e\in\mathrm{WST}^{\beta_n}(K_n)} U_e\right]$.

But most importantly, we study the global geometry of the model: we prove that the typical diameter of $\mathrm{WST}^{\beta_n}(K_n)$ grows like $\Theta(n^{1/3})$ for $\beta_n \geq n^{4/3+o(1)}$ likewise the $\mathrm{MST}(K_n)$ case, and it grows like $\Theta(n^{1/2})$ for $\beta_n \leq n^{1+o(1)}$ similarly to the $\mathrm{UST}(K_n)$ case. For $\beta_n = n^\alpha$ with $1 < \alpha < 4/3$, the behavior of the typical diameter is a more delicate open question, but we conjecture that its exponent strictly between $1/2$ and $1/3$.

Akshay Hegde (University of Oxford)
Limit theorems for high critical points of smooth Gaussian fields

Abstract: We consider the point process of local maxima of stationary smooth Gaussian fields in a box $[0,R]^d \subset \mathbb{R}^d$ exceeding a level $u(R)$.We show that this point process converges weakly, after suitable rescaling, to homogeneous Poisson point process when $u(R) \to \infty$ as $R \to \infty$. Previously, it was shown only at the level $u(R)$ of expected maxima ($\simeq \sqrt(2d\log R)$) of the field in the box $[0,R]^d$. Proof relies on the fact that simple point processes are characterised by avoidance probabilities, then approximating avoidance probabilities by excursion probabilities. We also consider the discrete process of high points of the fields on a lattice of mesoscopic size and try to approximate it with an independent Bernoulli process.

Armand Bernou (Université Lyon 1 Claude Bernard)
Uniform-in-time estimates for interacting Brownian particles and Gibbs relaxation

Abstract: This talk focuses on N-particle systems with mean-field interactions, following a Brownian or Langevin dynamics. Those corresponds to the underlying system leading to the McKean-Vlasov or Vlasov-Fokker-Planck equations, respectively. In the case of a smooth interaction potential, we derive uniform in time estimates for the many-particle correlation functions, with quantified size in N that fits the physical prediction. Our main tools are linear derivatives with respect to the measure, as introduced by Otto and Lions, and in particular Lions expansions in the context of interacting particle systems, Glauber calculus, and new ergodic estimates for kinetic equations to treat the Langevin dynamics. Those controls on the correlations

are a first step towards the justification of a uniform in time CLT, a concentration inequality, and also allows for a finer understanding of the cross mean-field/relaxation error quantifying the convergence towards the Gibbs measure. Joint work with Mitia Duerinckx (ULB).

Dominik Nowak (University of Basel)
The Lorentz Gas in the Weak Coupling Regime: Derivation of the Linear Landau-Vlasov Equation

Abstract: We study the dynamics of a test particle in a system of N randomly distributed stationary spherical obstacles (scatterers) in dimensions $d \geq 2$. We assume that the test particle's motion is influenced by two contributing factors. One contribution comes from collisions with scatterers, whose interaction potential is modelled by $\varepsilon^{\alpha} U(r/\varepsilon)$, where $\alpha \in (0, 1/2]$ and U is radially symmetric and strictly decreasing. The second factor is a long range force field of mean-field type generated by the collection of all scatterers in the system. Although such systems are well understood when considering either collisions or an external force field, studying these two driving forces simultaneously leads to a combination of both local and non-local effects, which introduces new technical difficulties. In the weak coupling regime, we prove that for $\alpha \in (0, (d-1)/8)$ the test particle's probability density converges to the solution of the linear Landau-Vlasov equation as $\varepsilon \to 0$.

This talk is based on joint work with Chiara Saffirio.

Grigory Terlov (Rényi Institute)
Random optimization problems at fixed temperatures

Abstract: We consider a class of disordered mean-field combinatorial optimization problems, focusing on the Gibbs measure, where the inverse temperature does not vary with the size of the graph and the edge weights are sampled from a general distribution. We prove Central Limit Theorems for the log-partition function, the weight of a typical configuration, and the Gibbs average in both quenched and annealed forms. We also derive quenched Poisson convergence for the size of the intersection of two independent samples, yielding the strong replica symmetry of the model. Applications cover popular models from the literature, such as the Minimal Matching Problem, Traveling Salesman Problem, and Minimal Spanning Tree Problem, on a sequence of deterministic and random dense block graphs of increasing size. Joint work with Partha S. Dey

William Verreault (University of Toronto)
Permanent of random Rademacher matrices

Abstract: We give a short and simple proof of a result of Tao and Vu, namely that the permanent of an $n \times n$ random matrix with independent ± 1 entries grows like $n^{n/2+o(n)}$ with high probability. Our approach also applies to more general random matrices and works for symmetric matrices, which supersedes recent work of Kwan and Sauermann. This is joint work with Matteo D'Achille.

Niklas Schubert (Ruhr-Universität Bochum)
On the extremal decomposition of A-localized states for clock models on trees

Abstract: We consider $\mathbb{Z}_q$-valued clock models on a regular tree. Recently it has been proven that, at strong enough coupling, families of homogeneous Markov chain Gibbs states μ_A coexist whose single-site marginals concen- trate on $A \subset \mathbb{Z}_q$, and which are not convex combinations of each other [Abbondandolo, Henning, Külske, Majer, 2024]. The key point of our work is to aim at a description of the extremal decomposition of μ_A into all extremal Gibbs measures, which may be spatially inhomogeneous. We show that in a low temperature regime the decomposition is supported on uncountably many inhomogeneous extremal states, that we call *glassy states*.

Thursday

Han Le (University of Michigan)
Overlaps in bipartite spherical SK model

Abstract: We consider two overlaps in the bipartite spherical SK model, with no external field: the overlap with a replica, and an absolute overlap with the ground state. In particular, we derive fluctuation results in the low temperature and the high temperature regimes for them. This is based on forthcoming work with Jinho Baik.

Haotian Gu (Duke University)
On maximum of Poissonian log-correlated fields

Abstract: In recent years there has been intense ineterest in extreme values of logarithmically correlated fields (LCFs), in connection with problems on Gaussian multiplicative chaos, random matrices, branching random walks, reaction-diffusion PDE, and L-functions in analytic number theory. The sharpest results are for Gaussian or nearly-Gaussian fields. On the other hand, characteristic polynomials of sparse random matrices give rise to LCFs with Poissonian tails. In an earlier work on permutation matrices, Cook and Zeitouni obtained the leading order of the maximum. I will discuss new refined results on the maximum for a related class of random trigonometric polynomials with Poissonian tails. We find the sub-leading order behavior is significantly different from the ubiquitous "Bramson correction" term for Gaussian LCFs, and can be modeled by a branching random walk with a randomly time-varying offspring distribution. Based on joint work with Nicholas Cook (Duke).

Annabell Gros (University of Bonn)
Microscopic analysis of branching Brownian motion with decreasing velocities

Abstract: The extremes of variable speed branching Brownian motion (BBM) exhibit a phase transition when the “speed function", which describes the time-inhomogeneous

variance, is the identity function. In this talk we study this transition more closely by choosing piecewise linear, concave speed functions converging to the identity function from above. We show that the logarithmic correction of the order of the maximum interpolates smoothly between the correction of standard BBM, $\frac{3}{2\sqrt{2}}\log(t)$, and the correction for BBM with piecewise linear speed functions. A key part of the proof is the precise control of the path of an extremal particle. Based on joint work with Alexander Alban, Anton Bovier and Lisa Hartung.

Lucas D'Alimonte (LPSM, Sorbonne-Universités, Paris, France)
Ornstein–Zernike theory for the two dimensional near-critical random cluster model

Abstract: In this talk, we will discuss the classical Ornstein–Zernike theory for the random-cluster models (also known as FK percolation). In its modern form, it is a very robust theory, the most celebrated output of which being the computation of the asymptotically polynomial corrections to the pure exponential decay of the two-points correlation function of the random-cluster model in the subcritical regime. We will present an ongoing project that extends this theory to the near-critical regime of the two-dimensional random-cluster model, thus providing a precise understanding of the Ornstein–Zernike asymptotics when p approaches the critical parameter p_c. The output of this work is a formula encompassing both the critical behaviour of the system when looked at a scale negligible with respect to its correlation length, and its subcritical behaviour when looked at a scale way larger than its correlation length. Based on a joint work with Ioan Manolescu.

Malo Hillairet (Institut Fourier, Grenoble, France) and **Ekaterina Toropova** (Stockholm University)
Noise sensitivity in spatial growth models

Abstract: Spatial growth models include First-passage percolation and Last-passage percolation, of which the main objects of interest are travel times and so-called geodesics, which are the paths corresponding to optimal travel times in a random environment. We study noise sensitivity in these models, which is the property that randomly modifying the environment by a small noise induces the travel times to decorrelate with what they were before applying the noise, in the limit of large scales. This is motivated by the recent work of Daniel Ahlberg and Daniel de la Riva, who proved noise sensitivity of the indicator function of being above the median for some modification of travel times in First-passage percolation. We extended noise sensitivity notion and results to real-valued functions instead of indicator functions in order to prove the noise sensitivity of the travel times themselves. Joint work with Daniel Ahlberg.

Oleksii Kolupaiev (Institute of Science and Technology Austria)
Loschmidt echo for deformed Wigner matrices

Abstract: We will discuss recent results on sensitivity of a quantum evolution to perturbations. Consider two self-adjoint Hamiltonians $H_1 \approx H_2$ and an initial quan-

tum state ψ_0. First, evolve ψ_0 under the Hamiltonian H_1 from time zero to t, and then consider the backward evolution from t to zero under H_2, resulting in ψ_0'. The question is how precisely the initial state is recovered. One possible way to measure this revival is the Loschmidt echo $|\langle\psi_0, \psi_0'\rangle|^2$. We model H_1, H_2 by deformed Wigner matrices and show that the Loschmidt echo follows a universal behavior as a function of time. Our proof relies on a two-resolvent global law, i.e. a concentration estimate for products of resolvents of H_1 and H_2.

Arianna Piana (Weizmann Institute of Science)

Tensorized Stochastic Localization processes and efficient sampling from tensor Ising models

Abstract: In recent years, Stochastic Localization has proven to be a valuable tool for analyzing measures, such as log-concave measures and Ising measures, by tilting them with a Gaussian factor. In this work, we extend the technique to incorporate non-Gaussian tilts and introduce a tensorized version of Stochastic Localization. We show its implications for spectral gap estimates and the fast mixing behavior of Glauber dynamics. Joint work with Dan Mikulincer.