Interfaces of the Theory of Combinatorial Limits Workshop
Abstracts

Ágnes Backhausz
Typical processes on the infinite regular tree and random regular graphs
Abstract: Graph limit theory provides various tools to understand the structure of random graphs. In the talk we focus on random regular graphs, whose limit is very well understood in the Benjamini–Schramm sense, but the question whether the converge in the local-global sense remains open. The notion of typical processes (invariant families of random variables indexed by the vertices of the infinite d-regular tree, which can be approximated with functions on the vertices of random d-regular graphs) is closely related to this quesion. In the talk we present a sufficient condition for a process to be typical, which is based on entropy, and which might lead to a better understanding of the structure of random d-regular graphs. Joint work with Charles Bordenave and Balázs Szegedy.

Ferenc Bencs
On the number of forests and trees in large regular graphs
Abstract: In this talk I plan to discuss the number of trees ($\tau(G) = T(G, 1, 1)$), forests ($F(G) = T(G, 2, 1)$) (and acyclic orientations $= T(G, 2, 0)$) in d-regular graphs. Specifically, we will investigate these quantities along (any) sequences of d-regular graphs of girth tending to infinite. As well we will see that the limit of $T(G_n, x, y)^{1/v(G_n)}$ exists, where $x \geq 1, 0 \leq y \leq 1$ and $T(G, x, y)$ is the Tutte-polynomial. Joint work with Péter Csikvári.

Endre Csóka
The structure of random regular graphs
Abstract: We examine the structures and phase transitions in large random graphs. For example, the structure of a largest independent set in a random d-regular graph on $n \to \infty$ vertices is in a "solid state" for $d \geq 20$ but probably in a "spin glass state" for $d \leq 19$. We are using the tools of graph limit theory, local algorithms (IID-factor processes), and probability theory, and we are trying to translate and extend some intuitive arguments in statistical physics into this mathematical language. I will give an overview of this research topic.

Frederik Garbe
Dense limit theories for discrete structures - Graphons, Permutons, and Latinons
Abstract: We want to discuss the general approach for introducing a limit theory for combinatorial objects in the dense setting. As examples for this we will introduce the limit notions for dense graphs and permutations as

well as the newly developed limit theory for Latin squares. The last notion is joint work with Robert Hancock, Jan Hladký and Maryam Sharifzadeh.

Viktor Harangi
Counting spanning forests, invariant percolations on regular trees, and the f-invariant
Abstract: We show how graph limit theory and Bowen's f-invariant can be used for counting problems over random d-regular graphs. We develop techniques for maximizing the f-invariant (an entropy-like quantity). As an application we describe the local behaviour of a typical spanning forest of given density over random regular graphs.

Jan Hladký
Flip processes
Abstract: We introduce a class of random graph processes, which we call *flip processes*. Each such process is given by a *rule* which is just a function $\mathcal{R} : \mathcal{H}_k \to \mathcal{H}_k$ from all labelled k-vertex graphs into itself (k is fixed). The process starts with a given n-vertex graph G_0. In each step, the graph G_i is obtained by sampling k random vertices $v_1, \ldots, v_k$ of G_{i-1} and replacing the induced graph $F := G_{i-1}[v_1, \ldots, v_k]$ by $\mathcal{R}(F)$. This class contains several previously studied processes including the Erdős–Rényi random graph process and the triangle removal process. Actually, our definition of flip processes is more general in that $\mathcal{R}(F)$ is a probability distribution on $\mathcal{H}_k$, thus allowing randomized replacements. Given a flip process with a rule $\mathcal{R}$, we construct time-indexed trajectories $\Phi : \mathcal{W}_0 \times [0, \infty) \to \mathcal{W}_0$ in the space of graphons. We prove that for any $T > 0$ starting with a large finite graph G_0 which is close to a graphon W_0 in the cut norm, with high probability the flip process will stay in a thin sausage around the trajectory $(\Phi(W_0, t))_{t=0}^T$ (after rescaling the time by the square of the order of the graph). These graphon trajectories are then studied from the perspective of dynamical systems. Among others, we study continuity properties of these trajectories with respect to time and the initial graphon, existence and stability of fixed points and speed of convergence (whenever the infinite time limit exists). We give an example of a flip process with a periodic trajectory. This is joint work with Frederik Garbe, Matas Šileikis and Fiona Skerman (arXiv:2201.12272). We also study several specific families flip processes. This is joint work with Pedro Araújo, Eng Keat Hng and Matas Šileikis (in preparation).

Aranka Hrusková
Limits of action convergent graph sequences with unbounded (p, q)-norms
Abstract: The recently developed notion of action convergence by Backhausz and Szegedy unifies and generalizes the dense (graphon) and local-global (graphing) convergences. This is done through viewing graphs as

operators and examining their dynamical properties. Suppose $(A_n)_n^\infty$ is a sequence of operators representing graphs, Cauchy with respect to the action metric. If $(A_n)_n^\infty$ has uniformly bounded (p, q)-norms where (p, q) is any pair in $[1, \infty] \times [1, \infty]$ except for $(\infty, 1)$, then Backhausz and Szegedy prove that $(A_n)_n^\infty$ has a limit which, moreover, must be self-adjoint and positivity-preserving. In the present work, we construct a large class of sequences whose only uniformly bounded (p, q)-norm is the $(\infty, 1)$-norm, but which converge nonetheless. We show that the limit objects in this case are not unique and need not be neither self-adjoint nor positivity-preserving.

Gábor Kun
TBA

Matthew Kwan
Large deviations in combinatorics
Abstract: I'll give a friendly introduction to large deviations in combinatorial settings, including, as time permits: (1) a brief summary of the groundbreaking work by Chatterjee and Varadhan which brought graph limits into the picture, (2) a taste of some of the most modern methods in the area, due to Kozma–Samotij and Harel–Mousset–Samotij, and (3) some interesting open questions on large deviations in Latin squares, related to my own work joint with Sah, Sawhney and Simkin.

Ander Lamaison
Quasirandomness in discrete structures
Abstract: In this talk we will present the concept of quasirandomness in discrete structures, as related to combinatorial limits. An important starting point in this theory is the 1989 paper of Chung, Graham and Wilson in which several conditions for graph sequences are given, all equivalent to having the uniform graphon as its limit. We will discuss similar results in other structures, such as hypergraphs and permutations.

Péter Pál Pach
Common two-equation systems over the two-element field
Abstract: A system of linear equations L over $\mathbb{F}_q$ is common if the number of monochromatic solutions to L in any two-colouring of $\mathbb{F}_q^n$ is asymptotically at least the expected number of monochromatic solutions in a random two-colouring of $\mathbb{F}_q^n$. In this talk we will discuss the case of two-equation systems over $\mathbb{F}_2$ and prove partial results towards characterising the common ones. Joint work with Dan Král' and Ander Lamaison.