Workshop on Cohomological and metric aspects of Hermitian and almost complex manifolds

Monday (September 08)

9:30-10:30 **Richard Hind: Symplectic geometry of some Reinhardt domains**

Abstract: We will discuss domains of the form $\{|w|^2 < (1+|z|^2)^{-p}\} \subset \mathbb{C}^2$ with $p > 1$. These are logarithmically convex but not biholomorphic to bounded domains, however, with the standard symplectic form, they are symplectomorphic to bounded domains but not to convex domains.

When $p < 2$ the domains admit symplectic, volume filling, embeddings only from manifolds whose boundaries have sufficiently large Minkowski dimension. In particular they are not the interiors of symplectic manifolds with smooth boundary, and give the first examples of the failure of symplectic packing stability.

This is joint work with Dan Cristofaro-Gardiner.

11:00-12:00 **Gueo Grantcharov: On the SL(n,H) and strong HKT geometries**

Abstract: In the talk we'll review the basics of the theory of hypercomplex manifolds and the geometry of two particular but complementary cases - when the manifold has trivial canonical bundle (the SL(n,H) case) and when it admits a strong HKT metric. The second part is based on a recent joint work with B. Brienza, A. Fino, and M. Verbitsky.

14:00-15:00 **Nicolina Istrati: The algebraic reduction of compact Vaisman manifolds**

Abstracts: Vaisman manifolds form a special class of Hermitian, non-Kähler manifolds. Their group of biholomorphisms contains a distinguished complex one-dimensional Lie subgroup G, which has compact closure H. When G=H, it is well known that the quotient of the Vaisman manifold by G is a projective orbifold, which is thus its algebraic reduction. I will explain how to obtain the algebraic reduction in the general setting as a quotient construction involving the group H.

15:30-16:30 **Daniela Angella: Some problems concerning canonical metrics in Hermitian non-Kähler geometry**

Abstract: We investigate several possible notions of "canonical" metrics that naturally arise in Hermitian non-Kähler geometry.

In particular, we study an analogue of the Yamabe problem in the non-Kähler setting, concerning the existence of Hermitian metrics with constant scalar curvature with respect to the Chern connection. We also develop a moment map interpretation of the Chern scalar curvature in the locally conformally Kähler setting. Another tool for highlighting "canonical structures" is the Chern–Ricci flow. The long-time behavior of its solutions is expected to reflect the underlying complex structure, and we present some evidence of this in the case of compact complex surfaces.

This talk is based on joint work with Simone Calamai, Mauricio Corrêa, Francesco Pediconi, Cristiano Spotti, Valentino Tosatti, and Oluwagbenga Joshua Windare.

Tuesday (September 09)

9:30-10:30 **Gil Cavalcanti:Generalized complex vs almost complex structures**

Abstract: The ultimate aim of this talk is to provide an example of an almost complex manifold which does not admit complex, symplectic or stable generalised complex structures and conjecturaly does not admit any generalised complex structure. In the talk we will introduce the concepts of generalized complex structure, stable generalized complex structure, explain the relevance of the example and outline the tools to prove our main claim. The talk is based on joint work with Bas Wensink.

11:00-12:00 **Valentino Tosatti: Regularity of the volume function**

Abstract: On a compact Kahler manifold, and on some non-Kahler ones, one can define the volume of a (1,1) cohomology class, which generalizes the classical concept of volume of a line bundle over a smooth projective variety. This way, one obtains a nonnegative real-valued function on $H^{1,1}$ which is strictly positive exactly on the big cone. I will discuss the problem of understanding the optimal regularity properties of this function, both in the interior and at the boundary of the big cone. Based on joint works with Filip and Lesieutre and with Cao.

14:00-15:00 **Lorenzo Sillari: Pseudoholomorphic vector bundles and the Kodaira dimension of almost complex manifolds**

Abstract: Pseudoholomorphic vector bundles are a generalization holomorphic vector bundles to the case when the base manifold is only almost complex. The study of their geometry, first introduced by de Bartolomeis and Tian (then continued by Lempert and Szoke, or Kruglikov), requires the interplay of the theories of Hermitian connections, of differential operators on smooth forms, and of Gromov's pseudoholomorphic curves. After reviewing the general theory of pseudoholomorphic vector bundles, with special attention to line bundles, we will focus on the canonical bundle of an almost complex manifold. This allows, following Chen and Zhang, to give a definition of plurigenera and Kodaira dimension of an almost complex manifold. These invariants are tightly linked with the existence of pseudoholomorphic fibrations on the underlying almost complex manifold, especially in real dimension 4. As an application of the theory, we will determine the Kodaira dimension of compact quotients of Lie group, and we will provide explicit computation techniques for it on pseudoholomorphic torus fibrations.

15:30-16:30 **Tat-Dat To: Uniform estimates for Green's Functions of Kähler metrics**

Abstract: In a remarkable series of works, Guo, Phong, Song, and Sturm obtained key uniform estimates for the Green's functions associated with certain Kähler metrics. These estimates yield uniform diameter bounds for Kähler metrics, requiring only an integral bound on the density of the volume form and no lower bound on the Ricci curvature. In this talk, we will explain their approach, broaden the scope of their techniques, and apply these results to the Kähler-Ricci flow and various families of canonical Kähler metrics. Finally, we will briefly discuss a similar problem in the non-Kähler setting.

16:30-17:30 **Nicoletta Tardini: Pluriclosed manifolds with parallel Bismut torsion**

Abstract: Several special non-Kähler Hermitian metrics can be introduced on complex manifolds. Among them, pluriclosed metrics deserve particular attention. They can be defined on a complex manifold by saying that the torsion of the Bismut connection associated to the metric is closed. These metrics always exist on compact complex surfaces but the situation in higher dimension is very different. We will discuss several properties concerning these metrics also in relation with the torsion of the Bismut connection being parallel. This is joint work with G. Barbaro and F. Pediconi.

Wednesday (September 10)

9:30-10:30 **Vestislav Apostolov: Non-Kähler Calabi-Yau geometries on three-folds:**

*Abstract:*In this talk I will discuss compact non-Kähler Hermitian manifolds whose Bismut-Ricci form is identically zero. These Hermitian manifolds have a canonical symmetry reduction to complex co-dimension one. In complex dimension three,the transverse geometry is actually Kähler and is then governed by a single 6th-order scalar PDE for the underlying Kähler metric. In the regular case, this leads to a special case of Dervan's Z-critical equations on a class of complex Kähler surfaces; furthermore, using that the corresponding formal symplectic form is positive definite, we obtain the complete tool-kit of GIT associated to the reduced geometry: Calabi-Matsushima-Licherowicz's theorem, Futaki's invariant, Mabuchi's functional, etc. In the special case when the scalar curvature of the reduced Kähler geometry is constant, we show that the Bott-Chern number $h^{1,1}_{BC} \geq 2$ with equality if and only if the metric is Bismut-flat, and hence a quotient of either $SU(2) \times \mathbb{R} \times \mathbb{C}$ or $SU(2) \times SU(2)$. This talk is based on a joint paper with Barbaro, Lee and Streets which is available on arXiv:2408.09648.

11:00-12:00 **Uros Kuzman: On "big" J-holomorphic discs**

Abstract: The theorem of Nijenhuis and Woolf is a fundamental result in almost complex geometry, stating that for any almost complex manifold (M, J), there exists a pseudoholomorphic curve through any point $p \in M$ in any tangent direction $v \in T_pM$. However, this theorem guarantees only small solutions to the generalized Cauchy–Riemann equation. In this talk, we will discuss approximation, deformation, and gluing techniques for "big" J-holomorphic discs, i.e., pseudoholomorphic maps from the unit disc into (M, J) whose images are not necessarily contained in a neighborhood of a single point. Moreover, we will present a method for constructing such big discs in Stein domains.

Thursday (September 11)

9:30-10:30 **Weiyi Zhang: Almost complex structures on ruled surfaces**

Abstract: In this talk, I will discuss recent progress on the geometry of ruled surfaces equipped with arbitrary tamed almost complex structures. Using techniques from Seiberg-Witten theory and the study of J-holomorphic subvarieties, I will explain how to construct homology classes that admit smooth J-holomorphic representatives for all tamed almost complex structures on irrational ruled manifolds, including the first such examples of non-spherical classes. I will then describe applications to symplectic geometry, focusing on two directions: the extension problem from cyclic to circle Hamiltonian actions (joint with N. Lindsay), and the constructions of almost Kähler forms via semipositive (1,1)-forms arising from the above mentioned non-spherical classes (based on the work of S. Cattalani).

11:00-12:00 **Kuang-Ru Wu: Mean curvature of direct image bundles**

Abstract: Let $E \to X$ be a vector bundle of rank r over a compact complex manifold X of dimension n. It is known that if the line bundle $O_{P(E^*)}(1)$ over the projectivized bundle $P(E^*)$ is positive, then $E \otimes \det E$ is Nakano positive by the work of Berndtsson. In this talk, we give a subharmonic analogue. Let $p : P(E^*) \to X$ be the projection and α be a Kähler form on X. If the line bundle $O_{P(E^*)}(1)$ admits a metric h with curvature Θ positive on every fiber and $\Theta^r \wedge p^*\alpha^{n-1} > 0$, then $E \otimes \det E$ carries a Hermitian metric whose mean curvature is positive.

As an application, we show that the following subharmonic analogue of the Griffiths conjecture is true: if the line bundle $O_{P(E^*)}(1)$ admits a metric h with curvature Θ positive on every fiber and $\Theta^r \wedge p^*\alpha^{n-1} > 0$, then E carries a Hermitian metric with positive mean curvature.

14:00-15:00 **Andrea Cattaneo:Automorphisms of Nakamura manifolds**

Abstract: In this talk we want to discuss how it is possible to describe the automorphism group of a Nakamura manifold. These manifolds are a kind of solvmanifolds which have been recently introduced, so we will give a brief recall of their construction first and then we will recollect some facts about their geometry (invariant forms, cohomology, Albanese map, ...). The main part of the seminar is dedicated to the description of the automorphism group of Nakamura manifolds, presenting a way to produce a "canonical form" for automorphisms and using them to construct other manifolds.

15:30-16-30 **Iulia Gorginian: The twistor space of a compact hypercomplex manifold is never Moishezon**

Abstract: A Moishezon manifold is a compact complex manifold that is bimeromorphic to a projective manifold. While the twistor space of compact hyperkahler manifolds is known to be far from being Moishezon, I will explain why this property also fails for the twistor space of compact hypercomplex manifolds.

Friday (September 12)

9:30-10:30 **Sławomir Kołodziej: Solutions of complex Hessian equations on Hermitian manifolds**

Abstract: This is joint work with Cuong Ngoc Nguyen. We develop a theory of weak solutions for complex Hessian equations on compact Hermitian manifolds which resembles pluripotential theory for plurisubharmonic functions.

11:00-12:00 **Misha Verbitsky: Degenerate twistor deformations are Kähler**

Abstract: Let $\pi : M \to X$ be a Lagrangian fibration on a holomorphically symplectic manifold. Adding a closed (1,1)-form to the holomorphic symplectic form Ω, we obtain a holomorphic symplectic form on a complex deformation of (M, Ω). This deformation is called "degenerate twistor deformation". In the hyperkähler case the degenerate twistor deformation can be obtained as a degeneration of twistor deformations. A limit of a sequence of Kähler manifolds is not necessarily Kähler, and for many years it was unknown whether the degenerate twistor deformation of a compact hyperkähler manifold is always Kähler. I will present a recent proof of this conjecture, based on Harvey-Lawson duality criterion of Kählerness. This is a joint work with Andrey Soldatenkov.