Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00 -8:50	Registration				
8:50-9:00	Opening				
9:00 -9:50	**K. Falconer**	**D-J. Feng**	**Z. Buczolich**	**B. Solomyak**	**M. Pollicott**
9:55 -10:30	**E. Mihailescu**	**H. Chen**	**Y. Demichel**	**J. Schmeling**	**F. Przytycki**
10:30 -11:00	coffee break	coffee break	coffee break	coffee break	coffee break
11:00 -11:50	**X. Jin**	**J. Fraser**	**A. Käenmäki**	**T. Keleti**	**M. Wu**
11:55-12:20	**H. Ren**	**A. Rutar**	**R. Anttila**	**A. Gáspár**	Lunch break
12:20 -14:30	Lunch break	Lunch break	Lunch break	Lunch break	
14:30 -15:20	**K. Taylor**	**M. Rams**	Excursion	**B. Chand**	Farewell
15:20 -15:50	coffee break	coffee break		coffee break	
15:50 -16:25	**R. Balka**	**S. Seuret**		**R. Miculescu**	
16:30-16:55	**A. Banaji**	**A. Pyörälä**		**A. E. Orellana**	
17:00-		Poster Session		Lightning Talks	

- Invited talks: $45 + 5$ minutes,
- Medium-length talks: $30 + 5$ minutes,
- Short-length talks: $20 + 5$ minutes,
- Lightning talks: 5 minutes.

Abstracts

Invited speakers:

Zoltán Buczolich (ELTE Eötvös Loránd University, Hungary)

Multifractal Functions

In this talk I will first survey several earlier results about multifractal properties of functions and after that I will say a few words about the developments related to the von Koch function.

The talk is based on several papers joint with J. Nagy and mainly with S. Seuret. The most recent part is based on a joint project with S. Seuret and Y. Demichel.

Bedabrata Chand (IIT Madras, India)

Fractal Multiquadric Approximant, Applications and Box Dimension Results

In this work, we introduce a novel self-referential fractal multiquadric (MQ) function that exhibits symmetry about the origin. The scaling factors are carefully constrained to maintain the differentiability and convexity of the traditional multiquadric function. As an application, we apply this function to solve an initial value problem involving irregular and nonlinear functions us- ing a collocation procedure. We also discuss the approximation properties and box-dimension results of fractal MQ approximants. Furthermore, by employing basis functions derived from translations of the fractal multiquadric function on a grid, we propose two fractal quasi-interpolants $L_C^\alpha f$ and $L_D^\alpha f$, designed to approximate both smooth and irregular functions. We analyze the convergence of $L_C^\alpha f$ and $L_D^\alpha f$ to f using uniform error estimates. Additionally, we investigate the properties of these quasi-interpolation operators, including their ability to reproduce linear polynomials, and their convexity, concavity, and monotonicity characteristics.

(Joint work with D. Kumar and P. R. Massopust)

Kenneth Falconer (University of St. Andrews, UK)

Fractal percolation on statistically self-affine sets

Benoit Mandelbrot introduced fractal percolation or 'curdling' as a statistically self-similar process based on a hierarchy of squares resulting in a random set F. With each square selected independently with probability p, Mandelbrot suggested that there was a critical probability p_c at which F undergoes a topological phase transition, changing as p increases through p_c from being totally disconnected to having non-trivial connected components. This was confirmed by Chayes, Chayes and Durrett who derived further properties of F, as did Dekking, Meester and others.

Analogously, one may consider percolation based on a self-affine set, starting with the unit square and repeatedly dividing rectangles into $m \times n$ subrectangles in the obvious way. With each rectangle selected with probability p this leads to a statistically self-affine set F. However, now there is a lack of symmetry between the horizontal and vertical directions and considerations of critical probabilities may become more awkward.

We will give an overview of fractal percolation and consider differences and similarities between the self-similar and self-affine cases. This is joint work with Tianyi Feng.

De-Jun Feng (The Chinese University of Hong-Kong, China)

Homogeneous iterated function systems with the weak separation condition

In this talk, I will present some partial results on the problem of characterising homogeneous iterated function systems on the line with attractor $[0, 1]$ and satisfying the weak separation condition. This problem is closely related to the study of spectra of numeration systems, Bernoulli convolutions and self-similar Delone sets. It is based on joint work with Ching-Yin Chan.

Jonathan Fraser (University of St. Andrews, UK)

The Assouad dimension analogue of the Falconer distance problem

The *Falconer distance problem* is a famous open problem in geometric measure theory. Roughly speaking, it asks us to understand the relationship between the dimension of the set of distances achieved between pairs of points in a Borel set and the dimension of the set itself. I will discuss the Assouad dimension version of this problem in the plane.

Xiong Jin (University of Manchester, UK)

On the size of intersections of self-similar sets

We shall use the projection theorem of Hochman and Shmerkin on product of self-similar measures, together with the percolation method to study the size of intersection of one-dimensional self-similar sets. A similar approach can also help studying the size of overlaps for one-dimensional self-similar sets with similarity dimension larger than 1.

Antti Käenmäki (Rényi Insitute of Mathematics, Hungary)

Thermodynamic formalism of countably generated self-affine sets

We further develop the thermodynamic formalism of affine iterated function systems with countably many transformations by showing the existence and extending earlier characterisations of the equilibrium states of finite affine iterated function systems to the countably infinite case. As an application, under mild conditions, we prove that the affinity dimension of a countable affine iterated function system is equal to the supremum of the affinity dimensions of its finite subsystems. We deduce corollaries concerning the Hausdorff dimension of countably generated self-affine sets in dimensions 1, 2, and 3 satisfying mild deterministic assumptions and in arbitrary dimension with generic translations. The talk is based on a joint work with Ian D. Morris.

Tamás Keleti (ELTE Eötvös Lóránd University, Hungary)

Lipschitz images, old and new dimensions

First we study the general problem if a given set or metric space A (or at least a subset of A) can be mapped onto another given set or metric space B by a Lipschitz map. Among others, we characterize those self-similar sets with the strong separation condition that can be obtained as the Lipschitz image of the Cantor set. We also give a characterization of those compact metric spaces that can be obtained as an α-Hölder image of $[0, 1]$. For the general case we show that if A and B are compact metric spaces and the Hausdorff dimension of A is greater than the upper box dimension of B then A has a subset that can be mapped onto B by a Lipschitz map.

As an application we show that in some sense every reasonable fractal dimension must be at least the Hausdorff dimension and at most the upper box dimension. We also study some variants of this result in which packing dimension, Assouad dimension and modified lower dimension also appear as greatest or smallest among dimensions with given natural properties.

We also study the problem of Fraser to characterize Hausdorff dimension as the unique dimension with given natural properties. To disprove a candidate we introduce a new family of fractal dimensions by restricting the set of diameters in the coverings in the usual definition of the Hausdorff dimension. Among others, it turns out that this family contains continuum many distinct dimensions, and they share most of the properties of the Hausdorff dimension. We also consider the supremum of these new dimensions, which turns out to be another interesting notion of fractal dimension.

Joint work with Richárd Balka.

Mark Pollicott (University of Warwick, UK)

Lapidus complex dimensions, Borthwick zeros and resonances

Lapidus introduced the notion of "complex dimensions" associated to Cantor sets in the real line (usually occurring as limit sets of iterated function schemes). These are a countable family of complex numbers, which includes the box dimension (hence the name). We shall compare these to the Borthwick zeros, another family of complex values this time associated to infinite area Riemann surfaces, and resonances, yet a third family of complex values associated to the long term behaviour of flows on hyperbolic sets.

Michał Rams (Polish Academy of Sciences Institute of Mathematics, Poland)

Constructing nonhyperbolic ergodic measures

I will present a construction, obtained with Lorenzo Diaz and Katrin Gelfert, of high entropy nonhyperbolic ergodic measures.

Boris Solomyak (Bar-Ilan University, Ramat-Gan, Israel)

Dimension properties of spectral measures for some dynamical systems and Lyapunov exponents of associated cocycles

We study the local dimension properties of spectral measures for some dynamical systems (substitutions, interval exchange transformations, translation flows on flat surfaces) and a related question: is the spectrum singular? One of the tools is a so-called "spectral cocycle", or "twisted cocycle", whose Lyapunov exponents play an important role. The talk will be a gentle introduction into this area, mostly based on a series of joint papers with A.I. Bufetov.

Krystal Taylor (The Ohio State University, USA)

Efficient coverings of fractal subsets of the plane and a prescribed projection result

A classic theorem of Davies states that a set of positive Lebesgue measure can be covered by lines in such a way that the union of the set of lines has the same measure as the original set. This surprising and counter-intuitive result has a dual formulation as a prescribed projection theorem. We investigate an analogue of these results in which lines are replaced by shifts of a sufficiently nice fixed curve.

In particular, we show that measurable subsets of the plane can be covered by translations of a fixed open curve, obeying some mild curvature assumptions, in such a way that the union of the translated curves has the same measure as the original set. Our results rely on a Venetian blind construction and extend to transversal families of projections. As an application, we consider how duality between curves and points can be used to construct nonlinear Kakeya sets.

Meng Wu (University of Oulu, Finland)

On normal numbers in fractals

Given any Bernoulli measure μ that is $\times 3$ invariant (such as the Cantor-Lebesgue measure on the ternary Cantor set) and an irrational number t, it holds that for almost all x with respect to μ, the product tx is $\times 3$ normal—meaning that the orbit of tx under the $\times 3$ map is uniformly distributed on $[0, 1]$. This nice result was recently proved by Dayan, Ganguly, and Barak Weiss using sophisticated techniques from random walk theory. We will present a new proof of the Dayan-Ganguly-Weiss result, utilizing recent advancements in the study of self-similar measures with overlaps. Our approach extends the result to cases where the measure μ is only required to be invariant, ergodic, and of positive dimension.

Medium talks:

Richárd Balka (HUN-REN Alfréd Rényi Institute of Mathematics, Hungary)

Prevalent homeomorphisms of $[0,1]^d$ *and fractal dimensions*

Prevalence is a measure theoretic concept, which generalizes the notion of "almost every" in non-locally compact Polish groups, where there exists no Haar measure. The group $\mathrm{Homeo}([0,1]^d)$ is not commutative, and the theory of Haar null sets is notoriously complex for non-abelian groups.

In the first part, I will discuss regularity properties of homeomorphisms. For example, a prevalent $h \in \mathrm{Homeo}([0,1]^d)$ is singular, namely it maps a full measure set into a set of zero measure. This implies that the graph of a prevalent $h \in \mathrm{Homeo}([0,1])$ is of length 2, answering a question of Mycielski. This is a joint work with Márton Elekes, Viktor Kiss, and Márk Poór.

In the second part, fractal dimensions of graphs of homeomorphisms will be considered from the point of view of prevalence. A new and interesting phenomenon is that there is no prevalent value for the dimension of the graph in case of the packing and box dimensions. The case of Hausdorff dimension remains open. Finally and most importantly, I will pose a number of problems.

Haipeng Chen (Shenzhen Techonology University, China)

Dimensions of the popcorn graphs

Popcorn functions (known as Thomae's function, Riemann function, stars over Babylon etc.) and their graph set are important in the research of real analysis, number theory, fractal geometry, and other related fields. In this talk, we will introduce a class of popcorn functions and their graph sets (called popcorn graphs),and show the recent progress in the dimension theory of the popcorn graphs and their variants, which mainly follow from the Duffin-Schaeffer type estimates from Diophantine approximation, and Chung-Erdős inequality from probability theory. These are joint works with Dr. Amlan Banaji, Prof. Jonathan Fraser, and Dr. Han Yu.

Yann Demichel (Université Paris Nanterre, France)

What can be said about the regularity of the von Koch function?

The Swedish mathematician Helge von Koch is known for constructing one of the first and most famous self-similar fractal curves, which gave rise to the iconic 'Snowflake Curve'. Much less well known, however, is von Koch's function, which he described at the end of his 1904 article. In this talk we will present the construction of this function, and look at its multifractal properties, namely the pointwise Hölder exponent and the Hausdorff spectrum of singularities. This is a work in progress in collaboration with Zoltán Buczolich (Eötvös Lóránd University, Budapest) and Stéphane Seuret (Université Paris-Est Créteil, Paris).

Radu Miculescu (Transilvania University of Brasov, Romania)

On the connectivity of graph Lipscomb's space

A central role in topological dimension theory is played by Lipscomb's space J_A. On the one hand, Lipscomb's space is the attractor of a possibly infinite iterated function system, i.e. it is a generalized Hutchinson-Barnsley fractal. As, on the other hand, some classical fractal sets are universal spaces, one can conclude that there exists a strong connection between topological dimension theory and fractal set theory. A generalization of Lipscomb's space, using graphs, has been recently introduced (see R. Miculescu, A. Mihail, Graph Lipscomb's space is a generalized Hutchinson-Barnsley fractal, Aequat. Math., **96** (2022), 1141-1157). It is denoted by $J_A^{\mathcal{G}}$ and it is called graph Lipscomb's space associated with the graph $\mathcal{G}$ on the set A. It turns out that it is a topological copy of a generalized Hutchinson-Barnsley fractal. This talk provides a characterization of those graphs $\mathcal{G}$ for which $J_A^{\mathcal{G}}$ is connected.

Eugen Mihailescu (Institute of Mathematics of the Romanian Academy, Romania)

Dimension for invariant measures for a class of skew product maps with singularities

I will present several recent results about the dimension theory and exact dimensionality of ergodic measures for certain classes of non-conformal skew product maps with singularities.

Feliks Przytycki (Institute of Mathematics of Polish Academy of Sciences, Poland)

Not many periodic trajectories in bunches for iteration of complex quadratic polynomials of one variable

I will sketch a proof that there are few periodic trajectories of any period, close to a Cremer periodic orbit. This is related to a geometric pressure defined via periodic orbits.

Jörg Schmeling (Lunds University, LTH, Center of Mathematics, Sweden)

Fast scaling of potentials with log-singularities

TBA

Stéphane Seuret (Université Paris Est Créteil, France)

Bivariate multifractal analysis - examples and counter-examples

TBA

Short talks:

Roope Anttila (University of Oulu, Finland)

Pointwise Assouad dimension for measures

I introduce a notion of pointwise dimension for measures, the pointwise Assouad dimension, which describes the extremal scaling behaviour of a measure at a given point. I discuss the relationship of the pointwise Assouad dimension to other more familiar notions of dimension with examples and explain the typical behaviour for classical measures such as for quasi-Bernoulli measures on self-conformal sets. Finally, if time permits, I will discuss the study of non-typical points for the pointwise Assouad dimension of self-similar measures via multifractal analysis. The talk is partially based on a joint work with Ville Suomala.

Amlan Banaji (Loughborough University, UK)

Fourier decay of fractal measures and their pushforwards

Determining when the Fourier transform of a measure decays to zero as a function of the frequency, and estimating the speed of decay if so, is an important problem. We will discuss this problem in relation to fractal measures arising from iterated function systems, explaining that systems with non-linearity often result in good decay. In particular, in joint work with Simon Baker we have used a disintegration technique to prove that the Fourier transform of non-linear pushforwards of a general class of fractal measures decay at a polynomial rate. Combining this with a result of Algom – Rodriguez Hertz – Wang and Baker – Sahlsten, we prove that for any IFS on the line consisting of analytic contractions, at least one of which is not affine, every non-atomic self-conformal measure exhibits polynomial Fourier decay.

Attila Gáspár (Eötvös Loránd University, Hungary)

1-*Lipschitz maps onto polygons*

Kolmogorov asked whether every set of finite measure in the plane can be mapped onto a polygon by a 1-Lipschitz map with arbitrarily small measure loss. The answer is negative in general, however, the question is still open for compact sets. We give a positive for sets with tube-null boundary. In particular, we prove that the Sierpiński carpet can be mapped into the union of finitely many line segments by a 1-Lipschitz map with arbitrarily small displacements, answering a question of Balka, Elekes and Máthé.

Ana Emilia de Orellana (St. Andrews, UK)

Projection theorems for the Fourier spectrum

The Fourier spectrum is a family of dimensions that interpolates between the Fourier and Hausdorff dimensions and helps us understand how they are connected. In this talk we will see how it proves useful to get sharper results for bounds regarding the Hausdorff dimension in situations where the Fourier dimension alone is not enough to capture the effect of the Fourier transform of measures. Joint work with Jonathan Fraser.

Aleksi Pyörälä (University of Jyväskylä, Finland)

On self-convolutions of fractal measures on the parabola

As a rule of thumb, convolving a measure with itself should substantially increase its dimension, subsequent convolutions even more so. I will discuss this phenomenon for fractal measures supported on the parabola: In a recent joint work with Tuomas Orponen and Carmelo Puliatti, we show that for any s-Frostman measure μ on the parabola and any $t < \min\{3s, s+1\}$, the dimension of the repeated self-convolution of μ is eventually bounded from below by t. This bound is sharp, and follows from a quantitative result on L^p-norms of Fourier transforms of such measures.

Haojie Ren (Department of Mathematics, Technion, Haifa, Israel)

The dimension of Bernoulli convolutions in $\mathbb{R}^d$

For $(\lambda_1, \ldots, \lambda_d) = \lambda \in (0,1)^d$ with $\lambda_1 > \cdots > \lambda_d$, denote by μ_λ the Bernoulli convolution associated to λ. That is, μ_λ is the distribution of the random vector $\sum_{n\geq 0} \pm(\lambda_1^n, \ldots, \lambda_d^n)$, where $\pm$ signs are chosen independently and with equal weight. Assuming for each $1 \leq j \leq d$ that λ_j is not a root of a polynomial with coefficients $\pm 1, 0$, we prove that the dimension of μ_λ equals $\min\{\dim_L \mu_\lambda, d\}$, where $\dim_L \mu_\lambda$ is the Lyapunov dimension. This is a joint work with Ariel Rapaport.

Alex Rutar (University of Jyväskylä, Finland)

Box dimensions of countably generated self-conformal sets

A general problem in the study of expanding dynamical systems is to understand to what extent invariance implies regularity. I will address a specific version of this general phenomenon: when does the box dimension of a countably generated self-conformal set exist? For the limit set Λ and taking F to be the orbit of some arbitrary point $x_0 \in \Lambda$, we have the immediate lower and upper bounds

$$\max\{\underline{\dim}_{\mathrm{B}} F, \dim_{\mathrm{H}} \Lambda\} \leq \underline{\dim}_{\mathrm{B}} \Lambda \leq \overline{\dim}_{\mathrm{B}} \Lambda.$$

It turns out, perhaps surprisingly, that the box dimension of Λ exists if and only if these lower and upper bounds coincide. In particular, this provides the first examples (as far as we are aware) of sets of continued fraction expansions with restricted digits for which the box dimension does not exist. This result follows from sharp asymptotics for the covering numbers $r \mapsto N_r(\Lambda)$ in terms of $\dim_{\mathrm{H}} \Lambda$ and the covering numbers $r \mapsto N_r(F)$, where $N_r(\cdot)$ denotes the least number of open balls of radius r required to cover a given set. This talk is based on joint work with Amlan Banaji.

Lightning talks:

Izabella Abraham (Transilvania University of Brasov, Romania)

Relational generalized iterated function systems

We introduce a wider class of generalized iterated function systems, called relational generalized iterated function systems. More precisely, the classical contraction condition for functions defined on product spaces is weakened by means of an equivalence relation. In particular, if we consider the total equivalence relation, we recover the classical generalized iterated function systems. Our main result states that each compact subset of the underlying metric space generates, via a sequence of iterates, a fixed point of the associated fractal operator, called an attractor of the system. We also establish a structure result for the attractors and a theorem concerning the continuous dependence of the attractor on the associated compact set. Ultimately, we provide some examples which illustrate our main results.

Aiswarya T (Indian Institute of Technology, Tirupati, India)

Nonlinear Histopolation Function

This work constructs a nonlinear fractal histopolation function associated with a given histogram. Since the iterated function system under examination consists of Rakotch contractions rather than Banach contractions, this work generalises the development of the fractal histopolation function.

Bogdan Anghelina (Transilvania University of Brasov, Faculty of Mathematics and Computer Science, Romania)

On the fractal operator of a mixed possibly infinite iterated function system

We introduce a new class of iterated function systems. More precisely, we study the fractal operator associated with a mixed possibly infinite iterated function system (briefly mIIFS). Such a system is a possibly infinite iterated function system (i.e. a possibly infinite family of Banach contractions, on a complete metric space, satisfying some extra conditions) enriched with an orbital possibly infinite iterated function system (i.e. a possible infinite family of nonexpansive functions which are not Banach contractions on the entirely previously mentioned complete metric space, but just on the orbits of space's elements). Our main result states that the fractal operator associated with a mIIFS is a weakly Picard operator whose fixed points are called attractors of the system. We present concrete examples of mIIFSs and graphical representations of certain attractors' approximants.

Ridip Medhi (IIT Delhi, India)

A Point-Fibred Iterated Function System Consisting of Local Radial Contractions

In the quest for an axiomatic IFS theory, it has been noted that favorable properties of the code map linked to an IFS facilitate deriving key conclusions of the classical Hutchinson-Barnsley theory. In this context, this note delves into the properties of the code map associated with an IFS composed of local radial contractions. Leveraging these established properties, the existence of a unique invariant set and invariant measure for the IFS is deduced, bypassing reliance on the properties of the hyperspace of compact subsets and the Hausdorff metric.

Yuhao Xie (The Chinese University of Hong Kong, China)

Dimensions of projections of typical self-affine sets

In fractal geometry, it is important to understand the fractal dimensions of self-affine sets. In 1988, Falconer introduced the concept of affinity dimension, and showed that the Hausdorff and box-counting dimensions of a typical self-affine set (in a natural sense) are equal to its affinity dimension. In this talk, I will present some results on the dimensions of orthogonal projections of typical self-affine sets. It is based on joint work with De-Jun Feng.