Workshop on Moduli of Curves

Abstracts

Arnaud Beauville: Maximal variation of linear systems

Abstract: Let X be a smooth projective complex variety, and L a line bundle on X . We say that the linear system —L— has maximal variation if its elements have the maximum number dim —L— of moduli. I conjecture that this is always the case if X is not uniruled and L is ample. I'll discuss some cases where the conjecture holds : hypersurfaces, K3 surfaces, hyperkähler manifolds, and abelian varieties.

Same in TeX: Let X be a smooth projective complex variety, and L a line bundle on X. We say that the linear system $|L|$ has maximal variation if its elements have the maximum number $\dim|L|$ of moduli. I conjecture that this is always the case if X is not uniruled and L is ample. I'll discuss some cases where the conjecture holds : hypersurfaces, K3 surfaces, hyperkähler manifolds, and abelian varieties.

Eduard Looijenga: Teichmüller potentials for tori and K3 surfaces.

Abstract: A fundamental result of Teichmüller theory states that if S and S' are Riemann surfaces of positive genius g , then any homeomorphism from S onto S' is isotopic to one which minimizes its quasiconformal dilatation. This homeomorphism is then unique unless g=1: is then unique up to translation. This minimum serves to define the Teichmüller distance on Teichmüller space of genus g.

We here begin a such a theory for K3 surfaces: we generalize this construction to flat n-tori and Ricci flat K3 manifolds (where in either case no compatible complex structure is assumed) and define among other things in these cases a Teichmüller distance which recovers the distance on their moduli spaces, when considered as symmetric spaces (for SL(n) and O(3,19) respectively). This is joint work with Benson Farb.

Dan Petersen: Stable cohomology of moduli spaces, and moments of L-functions

Abstract: This is a report of joint work with Bergström-Diaconu-Westerland and Miller-Patzt-Randal-Williams. The stable rational cohomology of M_g is known, by Madsen and Weiss's celebrated proof of the Mumford conjecture. One can similarly study the stable cohomology of M_g with coefficients in a symplectic local system; this was determined by Looijenga. We study instead the moduli space H_g of hyperelliptic curves. The cohomology with constant rational coefficients stabilizes for essentially trivial reasons, but the cohomology with symplectic coefficients is highly nontrivial. We compute the stable answer and prove an improved range of homological stability, by homotopical methods. These geometric results have strong consequences for arithmetic statistics over finite fields: our results prove the Conrey-Farmer-Keating-Rubinstein-Snaith predictions for all moments of the family of quadratic L-functions associated to hyperelliptic curves over F_q, for all sufficiently large fixed $_q$.

Soheyla Feyzbakhsh: Genus-12 K3 curves

Abstract: We show that a general genus-12 K3 curve C (which can be embedded into a K3 surface) lies on a unique Fano threefold of genus 12, which is isomorphic to a Brill–Noether locus of vector bundles on C. This is joint work in progress with Yiran Cheng.

Dawei Chen: The tautological ring of strata of differentials

Abstract: Differentials with prescribed zero orders stratify the Hodge bundle. These strata of differentials appear as important objects in moduli theory and surface dynamics. In this talk, I'll focus on the tautological ring of strata of differentials. In particular, we show cohomology stability for strata of differentials with sufficiently many simple zeros. Part of the talk is based on joint work with Hannah Larson.

Dave Jensen: The Kodaira Dimensions of Moduli Spaces of Curves

Abstract: We discuss the birational geometry of various moduli spaces, including moduli of curves, abelian varieties, and Prym varieties. After surveying the current state of research in this area, we will focus on recent work showing that the moduli spaces of curves of genus 22 and 23, and the moduli space of Pryms in genus 13 are of general type. These results use tropical methods to resolve specific cases of the Strong Maximal Rank Conjecture. This is joint work with Gavril Farkas and Sam Payne.

Martin Möller: Compactifying strata of abelian differentials, not just in characteristic zero

Abstract: Strata of abelian differentials are the moduli space of curves together with a differential form with prescribed orders of zeros and poles. We revisit the classification of components of strata of abelian differentials with an eye on arbitrary characteristic. As a main tool we construct a proper and smooth model over the integers away from a specific set of bad primes using a residue condition valid in mixed characteristic.

Laurent Manivel: On the birational type of some Fano fourfolds

Abstract: I will explain how the theory of Hodge atoms of Katzarkov-Kontsevich-Pantev-Yu can be applied to certain Fano fourfolds with properties similar to those of cubic fourfolds. Joint work with Benedetti, Fay, Guéré and Perrin.

Alastair Craw: Simultaneous resolution of the relative Hilbert scheme

Abstract: Every Kleinian singularity can be deformed to produce a flat family of symplectic surfaces, each of which is a Nakajima quiver variety, and the simultaneous resolution of these singular surfaces was first constructed by Brieskorn and studied further by Kronheimer. In this talk, I'll describe work in progress with Ryo Yamagishi where we construct a simultaneous resolution of the relative Hilbert scheme of this flat family of surfaces.

Andras Nemethi: Filtered lattice homology

Abstract: Lattice homology is a bigraded Z[U] module, it is a new invariant, which has different versions. Its topological version is associated with links of normal surface singularities, while its analytic version is associated with isolated complex analytic germs of dimension one or higher. It has an algebraic version as well: it is an invariant of finite codimensional integrally closed ideals in Noetherian k-algebras. Any version admits an improvement: the filtered lattice homology. The starting point is a canonical filtration, which induces a spectral sequence, whose pages code deep new information. We exemplify it using reduced curve singularities. In the plane curve cases it recovers the Heegaard Floer Link invariant of the link, but it has algebraic applications as well: e.g. it contains all the information about the Cohen-Macaulay finite/tame/wild type classification of curve germs.

Adam Gyenge: Functoriality of Logarithmic Hochschild Homology for Log Smooth Pairs

Abstract: Logarithmic Hochschild homology extends classical Hochschild invariants to log schemes via Artin fans, but establishing its unctoriality is hindered because logarithmic correspondences live on blow-up compactifications rather than ordinary products. To overcome the absence of a universal dg category of logarithmic sheaves and the failure of classical adjunctions, we introduce strong log

Fourier–Mukai kernels supported on certain canonical blow-ups and construct explicit unit- and counit-type adjunction data. We prove that logarithmic Hochschild homology is strictly functorial under the induced transforms, yielding a dg bicategory of logarithmic correspondences in which logarithmic Hochschild invariants act as categorical invariants. This framework also allows us to define logarithmic Chern characters and an associated logarithmic Euler pairing. Joint work with Márton Hablicsek and Leo Herr.

Phil Engel: Complex structures on S^6

Abstract: A recent manuscript written by Claude, and communicated by Levent Alpoge, constructs a 1-parameter family of compact complex threefolds diffeomorphic to S^6. I will describe the main geometric ideas behind the construction. Our goal is to emphasize the underlying geometry and the conceptual aspects of the proof, while avoiding heavy notations and computations whenever possible. I do not claim any originality of these results.

Bruno Klingler: Constructing algebraic cycles from variations of Hodge structures

Abstract: Given a smooth algebraic variety S over the complex numbers, I will explain how to construct natural finitely generated subrings of its Chow ring using Hodge theory.

Alina Marian: The full descendant intersection theory of Quot schemes on curves

Abstract:I will consider the intersection theory of the Quot scheme parametrizing sheaf quotients of arbitrary rank and degree of a trivial vector bundle on a smooth projective curve C. I will explain that all virtual tautological intersections on the Quot scheme are given by explicit compact formulas. The result generalizes on the one hand the decades-old Vafa-Intriligator formula, which calculates part of the tautological intersection theory. It also extends more recent formulas of Oprea and Pandharipande for the Segre integrals on the Quot scheme of zero-dimensional quotients. There are significant consequences for the intersection theory of the moduli space of stable bundles on C. The talk is based on ongoing joint work with Shubham Sinha.

Noah Arbesfeld: Crossed instantons in algebraic geometry

Abstract: In a series of papers, Nekrasov introduces moduli spaces of "crossed instantons," quiver representations that model instantons on unions of 2-planes in 4-space. The geometry of these spaces encodes information about moduli of sheaves on surfaces and produces deformations of characters of quantum affine algebras.

I will present joint work with Martijn Kool and Woonam Lim that studies Nekrasov's moduli spaces from the perspective of algebraic geometry. In particular, we explain how to define virtual invariants using a sheaf-counting theory for Calabi-Yau 4-folds. We also formulate a conjectural description of these moduli spaces as moduli of framed sheaves on a projective 4-fold.

Gergő Schefler: Lattice Homology of (certain) Artin algebras

Abstract: In this talk, we introduce a lattice homology theory for Artin k-algebras arising as quotients by integrally closed ideals. While the construction relies on choosing certain discrete valuations, our Independence Theorem shows that the resulting homology module is well-defined and its Euler characteristic equals the dimension of the algebra. A generalized version of this construction allows us to provide lattice homological categorifications for key invariants in singularity theory, e.g., the delta invariant and the geometric genus. Additionally, we present Newton diagrammatic interpretations of these invariants for quotients by monomial ideals of $k[x_1, x_2]$, as well as connections to annihilator ideals. This is joint work with A. Némethi.