Abstracts: Simons School on Moduli of Curves and K3 Surfaces

Rahul Pandharipande : Cycles on moduli spaces of bundles on curves

The lecture series will start with an elementary introduction to Abel-Jacobi theory on the Picard stack over the Artin stack of nodal curves. No understanding of these objects will be assumed. The first goal will be to arrive at Pixton's formula for the UniDR cycle (related to the closure of the zero section). The second half of the series will be about higher rank analogues of such cycles on the moduli spaces of bundles over the Artin stack of nodal curves.

Lucia Caporaso: Moduli spaces of Riemann surfaces and metric graphs.

The moduli theories of Riemann surfaces and of metric graphs have developed independently from one another for a long time, the former within algebraic and complex geometry, the latter within combinatorics and geometric group theory.

In the past 15 years they have been connected through tropical and non-Archimedean geometry, and today they are part of a broad picture that involves other interesting spaces, such as Teichmüller space and the Hurwitz space.

The course will introduce the two moduli theories, aiming to connect them together to illustrate the above picture with some of its applications.

Philip Milton Engel: Matroids and the Integral Hodge Conjecture

These lectures will describe joint work with de Gaay Fortman and Schreieder on the disproof of the integral Hodge conjecture for abelian varieties. The first lecture will develop the geometric background on toric varieties, abelian varieties, the Mumford construction, period maps, and monodromy. The second lecture will discuss regular matroids and their associated degenerations. We will state the main results and deduce some corollaries, including the stable irrationality of very general cubic threefolds. The third lecture will outline the first part of the proof: reduction to a combinatorial/linear algebraic problem. The final lecture will develop the remaining combinatorial ingredients of the proof, such as Albanese graphs and test functions, emphasizing both the conceptual and the computational aspects of the resulting problem.

Aaron Pixton: Universal Jacobians

Given a smooth genus g curve, its Jacobian is a g-dimensional abelian variety parametrizing degree 0 line bundles on the curve. This construction works in families, and when applied to the universal family over a moduli space of curves it gives the corresponding universal Jacobian. The goal of this course is to understand certain aspects of these universal Jacobians, focusing on their intersection theory. The universal Jacobian has a combination of features coming from two directions: curves (tautological classes, graph combinatorics) and abelian varieties (weight decomposition, Fourier transform).

We'll begin by introducing the tautological classes on the moduli space of smooth genus g curves and on symmetric powers of its universal curve. We'll then move from these symmetric powers to the universal Jacobian, loosely following work of Moonen, Polishchuk, and Yin. The second half of the course will discuss how to extend things over the moduli space of stable curves, using compactified Jacobians.