Workshop on singular canonical Kähler metrics on compact and noncompact manifolds

Tuesday (September 02)

9:30-10:30 **Christiano Spotti:Multiscale aspects of Kähler-Einstein metrics and Algebraic Geometry**

Abstract: The study of limits of Kähler–Einstein (KE) metrics reveals a rich multiscale structure, linking metric degenerations to diverse areas of Algebraic Geometry, including singularity theory, birational geometry, moduli theory, non-Archimedean geometry, and Hodge theory. It is reasonable to expect that the "analytic blow-up analysis" of such KE metrics—a differential-geometric approach to the regularity of solutions of this geometric PDE—may in fact be computable through algebro-geometric tools, sometimes even explicitly. Establishing such connections sheds new light on canonical algebraic structures, especially in the study of singularities and moduli spaces.

In this talk, I will discuss certain aspects of this story, aiming to describe some general framework based on the notion of scales. In the non-collapsing setting, I will focus on the algebro-geometric computability of metric bubbles' trees of asymptotically conical Calabi–Yau, possibly giving a differential geometric driven "resolution of singularities" of the non-collapsing part of the boundary of KE/K-moduli spaces. In the collapsing setting, I will discuss two (related) directions: (i) a conjectural unifying picture for algebraic compactifications of KE/K-moduli spaces (including the polarized Calabi–Yau case), and (ii) how the theory of collapsing Calabi–Yaus can also suggest an algebro-geometric interpretation of collapsing cuspidal regions in the negative KE case.

This talk is based on joint work and discussions with M. de Borbon, Y. Fang, Y. Odaka, and S. Sun.

11:00-12:00 **Annamari Ortu Stability of fibrations and perturbations of the base**

Abstract: On a fibration with smooth fibres, optimal symplectic connections are special metrics which solve a geometric PDE on the fibration. We study the existence of such metrics under perturbations of the Kähler class on the base, and we relate the existence of an optimal symplectic connection to an algebro-geometric stability condition on the fibration. To do this, we employ a perturbation technique based on a moment map interpretation of the optimal symplectic connection equation and the moment map flow. This is joint work (in progress) with Lars Martin Sektnan.

14:00-15:00 **Ye-Won Luke Cho: Continuity of singular Kahler-Einstein potentials**

Abstract: Eyssidieux-Guedj-Zeriahi showed in 2009 that any compact normal Kähler variety with trivial or ample canonical Q-line bundle admits a singular Kähler-Einstein (SKE) metric, generalizing the works of Aubin and Yau. A SKE potential generating the SKE metric is known to be locally Hölder continuous on the regular locus of the variety. But understanding the regularity of the potential on the singular locus remains as a major open problem. In this talk, we show that any SKE potential on a compact normal Kähler

variety is continuous on the variety. I shall also explain several geometric and analytic applications of the result. This is joint work with Young-Jun Choi.

15:30-16:30 **Yuchen Liu: Weighted K-moduli spaces**

Abstract: We construct compact moduli spaces for g-weighted K-polystable Fano varieties, which (by the YTD correspondence) are precisely Fano varieties admitting g-solitons. This includes the Kähler-Ricci solitons and Kähler-Ricci flat cone singularities as special cases. One of the main ingredients is a new proof of compactness for K-moduli spaces using birational geometry. Based on joint work with Harold Blum, Chenyang Xu and Ziquan Zhuang.

16:30-17:30 **Shih-Kai Chiu:Special Lagrangians from tropical curves**

Abstract: For any tropical curve G in $\mathbb{R}^n$ with unit edge weights, we construct a family of special Lagrangian submanifolds in T^*T^n, whose moment map images converge to G. This is based on a gluing construction that matches special Lagrangian local models to the combinatorics of G, thereby establishing a direct link between tropical geometry and special Lagrangian geometry. This is joint work with Yang Li and Yu-Shen Lin.