Programme

Conference "New structures in low-dimensional topology"

Sunday

1. 15:00 - 16:00

   JongHae Keum (Korea Institute for Advanced Study)

   TITLE: Fake Projective Planes

   ABSTRACT:

   A smooth compact complex surface with the same Betti numbers as the complex projective plane $\mathbb{P}^2$ is either $\mathbb{P}^2$ or is called a **fake projective plane(FPP)**. Indeed, a smooth compact complex surface with Betti numbers $b_0 = b_2 = b_4 = 1$, $b_1 = b_3 = 0$ has the top Chern number(Euler number) $c_2 = 3$, both geometric genus and irregularity 0, hence by Noether formula $c_1^2 = K^2 = 9$; thus Picard number = 1 and its canonical class K is either ample or anti-ample, and in the latter case the surface is isomorphic to $\mathbb{P}^2$. In other words, an FPP is a surface of general type with geometric genus $p_g = 0$ and $K^2 = 9$. Furthermore, it can be uniformized by it universal cover, the unit complex 2-ball, by Aubin and Yau, and its fundamental group is a co-compact arithmetic subgroup of $PU(2, 1)$ by Klingler. The existence of such a surface was first proved by Mumford in 1979, via 2-adic uniformization method. Algebraic varieties are not always described via polynomial equations: sometimes they are constructed via uniformization: this means, as quotients of bounded symmetric domains, via the action of discontinuous groups. General theorems (as Kodaira's) imply the algebraicity of these quotient complex manifolds. The problem concerning the algebro-geometrical properties of such varieties constructed via uniformization and especially the description of their projective embeddings (and the corresponding polynomial equations) lies at the crossroads of several allied fields: the theory of arithmetic groups and division algebras, complex algebraic and differential geometry, use of group symmetries, and topological and homological tools in the study of quotient spaces. Of particular importance are the so-called ball quotients, especially in dimension 2, since they yield the surfaces with the maximal possible canonical volume K^2 for a fixed value of the geometric genus p_g.

   In this talk I will report recent progress on FPPs, such as their derived categories, bicanonical maps and their equations.

- 16:30 - 17:30

  Discussion

- 18:00 - 20:00

  Dinner

Monday

1. 10:00 - 11:00

   Yongnam Lee (Korea Advanced Institute of Science & Technology)

   TITLE: Compact moduli of elliptic surfaces with a multiple fiber

   ABSTRACT: Motivated by Miranda and Ascher-Bejleri's works on compactification of moduli space of rational elliptic surfaces with a section, we study to construct compacti moduli space of elliptic surfaces with a multiple fiber. Particular emphasis is placed on the study of rational elliptic surfaces without section and Dolgachev surfaces. The main approach to understanding limit surfaces is Q-Gorenstein smoothing of slc surfaces. This is a joint work with Donggun Lee.

2. 11:30 - 12:30

   Kyungbae Park (Kangwon National University)

   TITLE: On lens spaces bounding smooth 4-manifolds with $b_2 = 1$

   ABSTRACT: In this talk, we address the question of which lens spaces can bound smooth 4-manifolds with second Betti number one under various topological conditions. Specifically, we show that there are infinite families of lens spaces that bound compact, simply-connected, smooth 4-manifolds with second Betti number one, yet cannot bound a 4-manifold consisting of a single 0-handle and 2-handle. Additionally, we establish the existence of infinite families of lens spaces that bound compact, smooth 4-manifolds with first Betti number zero and second Betti number one, but cannot bound simply-connected 4-manifolds with second Betti number one. The construction of such 4-manifolds with lens space boundaries is motivated by the study of rational homology projective planes with cyclic quotient singularities. This is joint work with Woohyeok Jo and Jongil Park.

3. 14:30 - 15:30

   Alexander Kubasch (Rényi Institute)

   TITLE: Introduction to the lattice cohomology of curve singularities

   ABSTRACT: The lattice cohomology of isolated curve singularities was introduced by T. Ágoston and A. Némethi in 2023. It is an embedded topological invariant of plane curves and analytic in higher codimensions. Similarly to the topological lattice cohomology of surfaces which can be thought of as an analytic version of Heegard Floer homology, the lattice cohomology of plane curves is closely related to Heegard Floer Link homology.

   In this introductory talk, I will define the lattice cohomology of a curve singularity, show numerous examples, and compare it to various other classical invariants, such as the delta invariant, the Seifert form, or the multivariate Poincaré series. I will also outilne how it detects both the Gorenstein property and the multiplicity via the notion of local minima. Joint work with A. Némethi and G. Schefler.

4. 16:00 - 17:00

   Gergö Schefler (Rényi Institute)

   TITLE: Lattice cohomology of curve singularities and beyond

   ABSTRACT: The analytic lattice cohomology of curve singularities has strong connection to other lattice cohomology theories and even Heegaard Floer Link theory. We will present how the weight function corresponding to irreducible plane curves can be computed, through a generalized Laufer sequence of universal cycles, from the weight function of the topological lattice cohomology corresponding to its minimal embedded resolution graph. This observation connects the embedded topology with the abstract analytic setup and also allows us to provide a new characterization of the Apéry set of the abstract semigroup of values in more geometric terms. These results are joint with A. Kubasch and A. Némethi.

The lattice cohomology of plane curve singularities is defined via valuations of the normalization. However, if the singularity is Newton nondegenerate, it is natural to use another set of valuations determined from the combinatorics of the Newton boundary. This provides a lattice cohomology with the same Euler characteristic, but with (usually) different weight functions. Our result with A. Némethi shows however, that the two lattice cohomologies agree. The methods allow us to extend the definition of the lattice cohomology to a more general algebraic setup, to certain ideals cut out by valuations having some special properties.

Tuesday

1. 10:00 - 11:00

   Cheolhyun Cho (Seoul National University)

   TITLE: On the mirror symmetry of invertible polynomials

   ABSTRACT: Berglund-Hübsch mirror symmetry is a duality between two invertible polynomials and their symmetry groups. In particular, in the case of curve singularities, geometric curves in the Milnor fiber of one singularity are related to the matrix factorizations of its transpose polynomial. We propose a geometric method to construct the mirror polynomial via Floer theory, and a canonical functor that transforms curves to matrix factorizations for curve singularities. This is a joint work with Choa and Jeong.

2. 11:30 - 12:30

   Dongwook Choa (Institute for Basic Science - Center for Geometry and Physics)

   TITLE: Unit normal correspondence of smooth divisor complemement

   ABSTRACT: A Liouville domain given by a complement of a Donaldson divisor has drawn much interest. We investigate a functor from a wrapped Fukaya category of the complement and a monotone Fukaya category of the divisor itself given by a unit normal correspondence. Then we consider a deformation of the wrapped Fukaya category by inverting a canonical Hamiltonian orbits of a circle action on the boundary. I will explain how the unit normal correspondence is extended to the deformation which turned out to be a quasi-isomorphism in favorable situations. Several examples will be discussed. This is an ongoing joint work with Hanwool Bae, Cheol-Hyun Cho, and Wonbo Jeong.

3. 14:30 - 15:30

   László Koltai (Rényi Institute)

   TITLE: Multiplier ideals of normal surface singularities

   ABSTRACT: We study the multiplier ideals, the corresponding jumping numbers and multiplicities $\{m(c)\}_{c\in\mathbb{R}}$ in the following context: (X,o) is a complex analytic normal surface singularity, $\mathfrak{a}\subset\mathcal{O}_{X,o}$ is an $\mathfrak{m}_{X,o}$–primary ideal, $\phi\colon \tilde{X}\to X$ is a log resolution of $\mathfrak{a}$ such that $\mathfrak{a}\mathcal{O}_{\tilde{X}}=\mathcal{O}_{\tilde{X}}(-F)$, for some nonzero effective divisor F supported on $\phi^{-1}(0)$. We show that $\{m(c)\}_{c>0}$ is combinatorially computable from F and the resolution graph Γ of ϕ, and we provide several formulae. We also extend Budur's result (valid for $(X,o)=(\mathbb{C}^2,0)$), which makes an identification of $\sum_{c\in[0,1]}m(c)t^c$ with a certain Hodge spectrum. In our general case we use Hodge spectrum with coefficients in a mixed Hodge module. We show that $\{m(c)\}_{c\le 0}$ usually depends on the analytic type of (X,o). However, for some distinguished analytic types we determine it concretely. E.g., when (X,o) is weighted homogeneous (and F is associated with the central vertex), we recover $\sum m(c)t^c$ from the Poincaré series of (X,o) and when (X,o) is a splice quotient then we recover $\sum m(c)t^c$ from the multivariable topological Poincaré (zeta) function of Γ.

4. 16:00 - 17:00

   Alex Hof (Rényi Institute)

   TITLE: Cohen-Macaulay Modules on Curves and Lattice Cohomology

   ABSTRACT: A Noetherian local ring containing a field admits at least one module with depth equal to the Krull dimension of the ring, called a maximal Cohen-Macaulay (MCM) module. A local ring is Cohen-Macaulay if and only if it is an MCM module over itself, and more broadly it is of interest to classify the MCM modules over a given ring. In particular, a ring is said to be of finite representation type if it admits only finitely many indecomposable MCM modules up to isomorphism; we will discuss how to characterize this property in the one-dimensional case using the lattice cohomology of the corresponding curve singularity. Based on joint work with András Némethi.

5. 18:00 - 20:00

   Dinner

Wednesday

1. 10:00 - 11:00

   DongSeon Hwang (Institute for Basic Science - Center for Complex Geometry)

   TITLE: Cascade structures on rational QHPPs

   ABSTRACT: Despite their fascinating applications such as the algebraic Montgomery—Yang problem, the classification of QHPPs remains an open challenge. Rational QHPPs and non-rational QHPPs exhibit significantly different properties, particularly in the abundance or absence of rational curves.

   In this talk, I will begin with a brief review of the classification of QHPPs, followed by a discussion on the classification of rational QHPPs by introducing the concept of cascades. In particular, all QHPPs with an ample anticanonical divisor can be classified. I will conclude by proposing the cascade conjecture, which, if true, would allow us to classify all rational QHPPs in terms of the cascades. In particular, the cascade conjecture implies the algebraic Montgomery—Yang problem. If time permits, some additional applications will also be mentioned.

2. 11:30 - 12:30

   Woohyeok Jo (Seoul National University)

   TITLE: On rational homology projective planes with quotient singularities of small indices

   ABSTRACT:

   In this talk, we study the effects of topological and smooth obstructions on the existence of rational homology complex projective planes that admit quotient singularities of small indices. In particular, we provide a classification of the types of quotient singularities that can be realized on rational homology complex projective planes with indices up to three, whose smooth loci have trivial first integral homology group. This is joint work with Jongil Park and Kyungbae Park.

3. 14:30 - 19:00

   Free afternoon

Thursday

1. 10:00 - 11:00

   Dongsoo Shin (Chungnam National University)

   TITLE: Deformations of sandwiched surface singularities and the minimal model program

   ABSTRACT: We study deformations of sandwiched surface singularities using the (anti-)Minimal Model Program (MMP) approach. First, we examine the relationship between various theories of deformations of sandwiched surface singularities, primarily through the framework of the Minimal Model Program. This work extends the results of Némethi and Popescu-Pampu on cyclic quotient surface singularities to a broader context. Next, we focus on Kollár's Conjecture, which claims that deformations of rational surface singularities are induced by their P-modifications. Using the anti-MMP method, we prove this conjecture for most sandwiched surface singularities. Parts of this presentation are based on joint works with Jaekwan Jeon and Heesang Park.

2. 11:30 - 12:30

   Seung-Jo Jung (Jeonbuk National University)

   TITLE: Tjurina spectrum for a hypersurface singularity

   ABSTRACT: The Steenbrink spectrum is a well-known invariant for a hypersurface isolated singularity. This talk introduces the Tjurina spectrum and compares it with the Steenbrink spectrum. More precisely, this talk shows that their difference has a canonical graded symmetry. Moreover, we see that using this we can improve the estimate of Briançon-Skoda exponent in some cases.

3. 14:30 - 15:30

   András Némethi (Rényi Institute)

   TITLE: Milnor fibers of sandwiched singularities

   ABSTRACT: Let us fix the topological type of a sandwiched complex normal surface singularity, and we consider all the possible smoothings of all analytic realizations. We prove that the number of such Milnor fibers (identified up to diffeomorphism) is finite.

4. 16:00 - 17:00

   Discussion

Friday

1. 10:00 - 11:00

   Ju A Lee (Seoul National University)

   TITLE: Lefschetz pencils on $\mathbb{CP}^2$ from a topological point of view

   ABSTRACT: Any projective surface admits a Lefschetz pencil, and more generally every symplectic 4-manifold admits a Lefschetz pencil and vice versa. Moreover, a Lefschetz pencil can be characterized by its monodromy factorization in the mapping class group of the fiber surface. In this talk, I'd like to introduce the recent progress about the topological construction of Lefschetz pencils on a complex projective plane.

2. 11:30 - 12:30

   Kyoung-Seog Lee (Pohang University of Science and Technology)

   TITLE: Instanton Floer homology and Milnor Fibers

   ABSTRACT: Recently there have been increasing interactions between low dimensional topology and algebraic/complex geometry. In this talk, I will discuss some of these interactions focusing on Floer homologies of links and algebraic/complex geometry of Milnor fibers of Brieskorn-Hamm complete intersection singularities. This talk is based on a joint work with Anatoly Libgober and Nikolai Saveliev.