Optimal Transport on Quantum Structures
– School –

Book of Abstracts

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Quantum Optimal Transport: dynamics

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Lecture 1, Quantum Entropy Inequalities: Inequalities for quantum entropy and Fisher information play an important role in the subject we shall discuss, and the first lecture is devoted to a comprehensive treatment of results that we shall use, including complete proofs. One of the key results is the Data Processing Inequality, which is one of the many equivalent forms of the Strong Subadditivity of Quantum Entropy, first proved by Lieb and Ruskai in 1973. This inequality says that the quantum relative entropy between any two quantum states is decreases under any quantum operation; that is, completely positive and trace preserving map.

Lecture 2, Reversible Quantum Markov Semigroups as Gradient flow for Quantum Relative Entropy: According to the Data Processing Inequality, for any semigroup of completely positive trace preserving operators; that is a quantum semigroup, the relative entropy of a state evolving under the action of this semigroup with respect to an invariant state is monotone decreasing. When can the semigroup be viewed as gradient flow with respect to some metric and the relative entropy? This is not always the case. We discuss the necessary conditions and the known sufficient conditions, giving details of the construction of the metrics. Many people have contributed to this subject, and while other approaches will be discussed, the focus will be on results obtained together with Jan Maas.

Lecture 3, Applications of the Gradient Flow Method: There are many motivations for writing Quantum Markov Semigroups in terms of gradient flow. One is that this provides a powerful means of proving certain useful functional inequalities, a point of view pioneered in the classical setting by Felix Otto. This approach is very geometric and curvature bounds for the metrics constructed in the second lecture play a fundamental role. In the third lecture we shall discuss a number of applications and also a number of open problems.

An introduction to classical optimal transport

Alessio Figalli
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In these three lectures, I will give a quick introduction to the classical optimal transport problem. I will discuss the connection between optimal transport and gradient flows via the Riemannian structure of the Wasserstein space, and I will also present some selected applications of this beautiful theory.

Quantum Optimal Transport: quantum channels and qubits

Dario Trevisan
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Several notions of distances based on optimal transport that are classically equivalent give rise instead to very different metrics in quantum settings. Aim of these three lectures is to describe two recently proposed approaches:

- a static (Monge-Kantorovich) formulation of the optimal transport problem, using quantum channels as the analogue of couplings (jointly with G. De Palma, "Quantum optimal transport with quantum channels", in Annales Henri Poincaré. Vol. 22, No. 10, 2021),
- a Wasserstein distance of order 1 for systems of qubits (or more general product systems), generalizing the classical optimal transport based on the Hamming metric (jointly with G. De Palma, M. Marvian and S. Lloyd, "The quantum Wasserstein distance of order 1", in IEEE Transactions on Information Theory 67.10, 2021).

We will highlight general properties of these distances and some applications, including Gaussian concentration inequalities and a modulus of continuity of von Neumann entropy with respect to the transport distance, as well as mention several open problems.

Plan of the lectures:

1. The Wasserstein distance of order 1: systems of qubits, definition of the distance and its properties, quantum Lipschitz constant and dual formulation, quantum channels and contraction coefficients.

2. Quantum Marton inequality and Gaussian concentration inequalities, modulus of continuity of the von Neumann entropy with respect to the Wasserstein distance of order 1.

3. Quantum optimal transport with quantum channels: quantum couplings, quadratic quantum optimal transport cost and main properties, optimal transport of quantum Gaussian states and quantum Gaussian channels.

Quantum Optimal Transport: Quantum couplings and Many-Body Problems

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Lecture 1:
Fundamental notions of classical optimal transport (Wasserstein distances of exponents 1 and 2, duality, Knott-Smith and Brenier theorems).
Basic functional analytic notions (trace-class and Hilbert-Schmidt operators, density operators).
Connes distance in noncommutative geometry.
Quantum extension of the Wasserstein distance of exponent 2.
Basic estimates and examples of computations.

Lecture 2:
Some applications of the quantum Wasserstein pseudometric:
(1) Mean-field and classical limits for quantum mechanics,
(2) Uniform in h convergence of time-splitting schemes for quantum dynamics,
(3) Observation inequalities for quantum dynamics.

Lecture 3:
Restricted triangle inequality for the quantum Wasserstein pseudometric.
Applications of the restricted triangle inequality (e.g. classical limit of the quantum pseudometric).
Quantum Kantorovich duality.
Applications of the duality to the triangle inequality.
Optimal transport from classical to quantum densities.