Focused Workshop on Quantum Rényi Divergences

Erdős Center, Budapest, 22-26 July 2024

Program

<table>
<tr><th></th><th>Monday 22.07</th><th>Tuesday 23.07</th><th>Wednesday 24.07</th><th>Thursday 25.07</th><th>Friday 26.07</th></tr>
<tr><td>10:00 – 11:15</td><td>Marco Tomamichel</td><td>Mario Berta</td><td>Anna Jenčová</td><td>Li Gao</td><td>Miklós Pálfia</td></tr>
<tr><td>11:15 – 11:45</td><td colspan="5">Coffee break</td></tr>
<tr><td>11:45 – 13:00</td><td>Nilanjana Datta</td><td>Omar Fawzi</td><td>Frits Verhagen
Erkka Haapasalo</td><td>Fumio Hiai</td><td>Yongdo Lim</td></tr>
<tr><td>13:00 – 14:30</td><td colspan="5">Lunch</td></tr>
<tr><td>14:30 – 15:15</td><td>Free discussion</td><td>Aadil Oufkir</td><td></td><td>Free discussion</td><td></td></tr>
<tr><td>15:15 – 16:00</td><td colspan="4">Free discussion</td><td></td></tr>
<tr><td>16:00 – 16:30</td><td colspan="4">Coffee break</td><td></td></tr>
<tr><td>16:30 – 18:00</td><td colspan="4">Free discussion</td><td></td></tr>
</table>

Conference dinner:

Time: Wednesday, 24 July, 19:00-22:00
Location: **Vígvarjú Restaurant**, 1051 Budapest, Vigadó tér 2.
google map

Talks

Speaker: Marco Tomamichel

Title: Conditional Rényi entropies from α to z and beyond

Abstract: This work in progress is an attempt to bring a little bit more order into the entropy zoo. We start with the observation that general α-z conditional entropies as defined so far are not closed under the entropic duality relation which relates conditional entropies for pure states. We solve this problem by introducing another parameter and thus a much larger class of conditional entropies that satisfy data-processing and additivity, and is closed under duality. We hope that this perspective eventually will lead to complete characterisations of conditional Rényi entropies and tighter entropic inequalities, but here I just report on some initial steps.

Speaker: Nilanjana Datta

Title: A curious lemma and a uniform continuity bound for the quantum relative entropy

Abstract:

Speaker: Mario Berta

Title: Variational formulae for relative entropies and applications

Abstract: I will discuss variational characterizations of locally-measured Rényi divergences for multipartite quantum states and corresponding applications in quantum information theory. My talk is partly based on arXiv:2304.14878 with Marco Tomamichel as well as arXiv:2405.05037 with Tobias Rippchen and Sreejith Sreekumar.

Speaker: Omar Fawzi

Title: Information capacities of quantum dynamical systems

Abstract: I will discuss the information transmission capabilities (both classical and quantum) of quantum dynamical systems. I will mostly focus on the infinite time setting and show that capacities have simple algebraic characterisations and behave particularly well in terms of additivity under tensor product and computability. Based on joint work with Mostafa Taheri and Mizanur Rahaman.

Speaker: Aadil Oufkir

Title: Non-signaling coding error exponents for classical-quantum channels

Abstract: In Shannon theory, it is well known that in order to send information with a rate strictly less than the channel's capacity the error probability can be exponentially small with the number of the channel uses. The rate of the decay is called error exponent or reliability function. The best known achievability error exponent is given by the random coding bound. On the other hand, the best known converse error exponent is given by the sphere packing bound. These bounds coincide for rates close to the capacity. We show that the sphere packing bound characterizes also the achievability error exponent for coding over classical-quantum channels with non-signaling strategies. This bound has no critical rate and is termed using Petz Rényi divergence in contrast to the plain coding where the right Rényi divergence is in general still unknown even above the critical rate.

Speaker: Anna Jenčová

Title: On $\alpha - z$-Rényi divergences in von Neumann algebras

Abstract: The $\alpha - z$-Rényi divergences form a family of quantum extensions of the classical Rényi divergences, which are fundamental in information theory. This family interpolates between the two well established quantum extensions: the Petz-type and the sandwiched Rényi divergences. Recently, the $\alpha - z$-Rényi divergences were extended to the general setting of von Neumann algebras. In this talk, some properties of this extension will be shown, with focus on the data processing inequality and reversibility of quantum channels. Reversibility and equality in DPI will be also discussed in the case when only positivity of the maps is assumed. The talk is based on a joint work with Fumio Hiai.

Speaker: Frits Verhagen

Title: Large-Sample and Catalytic Majorization and Multipartite Divergences

Abstract: A set of d probability distributions, respectively quantum states, $\varrho_1, \ldots, \varrho_d$ is said to majorize another set $\sigma_1, \ldots, \sigma_d$ if there exists one single stochastic map, respectively quantum channel, T such that $T\varrho_k = \sigma_k$ for all $k = 1, \ldots, d$. In this work, we consider the large-sample and catalytic cases, i.e. we ask whether there exists T such that for all k we have $T\varrho_k^{\otimes n} = \sigma_k^{\otimes n}$ for a sufficiently large number n of copies of the states, or $T(\tau_k \otimes \varrho_k) = \tau_k \otimes \sigma_k$ for some catalysts τ_k. In the case of finite probability distributions, we derive sufficient and almost necessary conditions for this to happen. These conditions are formulated in terms of multipartite monotones that generalize the well-known bipartite Rényi α-divergences. For each possible way that the supports of the d distributions can overlap relative to each other, there is a different family of such monotones involved in the conditions for majorization.

Our methods rely on an application of the Vergleichsstellensätze, a mathematical framework recently developed by T. Fritz. For quantum states, we have partial results that can be used to find *sufficient* conditions for large-scale and catalytic majorization for a pair of quantum states to a pair of probability distributions or vice versa. Finding conditions that are also *necessary* in these cases, or more generally identifying the monotones involved in the conditions for majorization for any number d of quantum states, is still an open problem even for $d = 2$. This problem is closely related to finding all quantum generalizations of the Rényi α-divergences.

Speaker: Li Gao

Title: Strong data processing inequality of GNS-symmetric quantum channels

Abstract: Quantum relative entropy is a fundamental measure in quantum information theory. The key property behind its wide applications is the monotonicity over quantum channels, also called data processing inequality. It indicates that two quantum states cannot be more distinguishable after the action of a quantum channel. In this talk, I will present an improved data processing inequality for GNS-symmetric quantum channel. As an application, this gives a tight relative entropy decay rate of quantum Markov semigroups. This talk is based on an joint work with Marius Junge, Nicholas LaRacuente and Haojian Li.

Speaker: Fumio Hiai

Title: Matrix quasi-arithmetic-geometric mean inequalities

Abstract: We consider quasi modifications of several operator mean functions of arithmetic and geometric type such as

$$\begin{aligned}
\mathcal{A}_{\alpha,p}(A,B) &:= (\alpha A^p + (1-\alpha)B^p)^{1/p}, \quad \text{quasi-arithmetic mean,} \\
R_{\alpha,p}(A,B) &:= (B^{\frac{1-\alpha}{2}p} A^{\alpha p} B^{\frac{1-\alpha}{2}p})^{1/p}, \quad \text{quasi-Rényi mean,} \\
G_{\alpha,p}(A,B) &:= (B^{p/2}(B^{-p/2}A^pB^{-p/2})^{\alpha}B^{p/2})^{1/p}, \quad \text{quasi-geometric mean,} \\
SG_{\alpha,p}(A,B) &:= ((B^{-p}\#A^p)^{\alpha}B^p(B^{-p}\#A^p)^{\alpha})^{1/p}, \quad \text{quasi-spectral geometric mean,} \\
LE_{\alpha}(A,B) &:= \exp(\alpha \log A + (1-\alpha)\log B), \quad \text{Log-Euclidean mean,}
\end{aligned}$$

for any $\alpha > 0$ and $p > 0$ and for positive (semi-)definite matrices A, B. For a given pair (M, N) from $\{\mathcal{A}, R, G, SG, LE\}$ we pursue the inequalities between $M_{\alpha,p}$ and $N_{\alpha,q}$ for $p, q > 0$ as well as between $M_{\alpha,p}$ and $M_{\alpha,q}$ for $p \neq q$, with respect to the different types of orders such as the Loewner order $M_{\alpha,p}(A,B) \leq N_{\alpha,q}(A,B)$, the entrywise eigenvalue order $\lambda(M_{\alpha,p}(A,B)) \leq \lambda(N_{\alpha,q}(A,B))$, the log-majorization $M_{\alpha,p}(A,B) \prec_{\log} N_{\alpha,q}(A,B)$, and the weak majorization $M_{\alpha,p}(A,B) \prec_w N_{\alpha,q}(A,B)$. Our final goal is to obtain the necessary and sufficient condition on α, p, q under which the inequality holds true for any positive definite matrices A, B when a pair (M, N) and a type of order are given, while the present situation is far from completion. Although this talk has the characteristics of pure matrix analysis, yet it might hopefully gain applications in quantum information, for instance, from the fact that $R_{\alpha,1/z}$ is the kernel operator function in defining α-z-Rényi divergences.

Speaker: Miklós Pálfia

Title: Law of large numbers for generalized operator means

Abstract: In this talk we investigate zeros of nonlinear operators in a Thompson metric space. Inspired by the work of Gaubert and Qu from 2014, we study exponentially contracting continuous and discrete time flows generated by these nonlinear operators. We establish the operator norm convergence of deterministic and stochastic resolvent and proximal type algorithms, in particular versions coming from a Trotter-Kato type formula. This generalizes recent strong law of large numbers and so called 'nodice' results proved for the Karcher mean of positive operators by Lim and Pálfia. Applications include generalization of these results from the Karcher mean to other, so called generalized Karcher means introduced in 2016. The talk is based on recent joint work with Zoltán Léka.

Speaker: Yongdo Lim

Title: Strong Convexity of Sandwiched Entropies and Related Optimization Problems

Abstract: We present several theorems on strict and strong convexity for sandwiched quasi-relative entropy (a parametrised version of the classical fidelity). These are crucial for establishing global linear convergence of the gradient projection algorithm for optimization problems for these functions. The case of the classical fidelity is of special interest for the multimarginal optimal transport problem (the n-coupling problem) for Gaussian measures.