POSTER TITLES AND ABSTRACTS

Workshop on statistical properties of chaotic dynamics in and out of equilibrium
Erdős Center, Budapest, 22 August 2024

01. Jason Atnip: Quasi-limit theorems in dynamical systems

In this poster we introduce and discuss several notions of limiting measures conditioned on surviving sets as well as their statistical properties. In particular, we show that for random dynamical systems enjoying the (random) spectral gap property, the random open invariant measure is the conditioning limit of the quasi-stationary measure. In addition, we show that the open invariant measure satisfies the quasi-ergodic property and we prove a strong law of large numbers and central limit theorem for conditioned measures.

02. Bernat Bassols Cornudella: *Conditioned stochastic stability of equilibrium states on uniformly expanding repellers*

We propose a notion of conditioned stochastic stability of invariant measures on repellers: we consider whether quasi-ergodic measures of absorbing Markov processes, generated by random perturbations of the deterministic dynamics and conditioned upon survival in a neighbourhood of a repeller, converge to an invariant measure in the zero-noise limit. Under suitable choices of the random perturbation, we find that equilibrium states on uniformly expanding repellers are conditioned stochastically stable. In the process, we establish a rigorous foundation for the existence of "natural measures", which were proposed by Kantz and Grassberger in 1984 to aid the understanding of chaotic transients.

03. Giovanni Canestrari: *Uniformly global observables for 1D maps with an indifferent fixed point*

Global-local mixing is a recent definition which aims at describing correlations between a local observable – an L^1 function – and a global observable – an L^∞ function which admits some kind of infinite volume average. By introducing “uniformly" global observables and cones of local observables with small logarithmic derivative, we study the problem of correlations for 1d-maps with an indifferent fixed point preserving an infinite measure, both on the interval and on the half-line, and we show global-local mixing for two general classes of such maps.

04. Raul S. Chavez: *Historical behavior of skew products and arcsine laws*

We consider skew products over Bernoulli shifts, where the fibered dynamics are given by diffeomorphisms of the interval. We study the historical behavior, referred to as non-convergence, of the Birkhoff average. We establish a connection between historical behavior and the arcsine law, which allows us to construct large classes of dynamics that provide an affirmative solution to Takens' last problem. These classes include one-dimensional dynamics such as Thaler's interval maps, and skew products whose interval fiber maps have a zero Schwarzian derivative or follow a random walk.

05. Wentao Fan: *Uniform Hyperbolicity of Asymmetric Lemon Billiard Section Map*

In the 1960s, Sinai discovered the dispersing mechanism when studying dispersing billiards. Then In 1970s, Bunimovich discovered the de-focusing mechanism for the hyperbolicity of the famous stadium billiards.
From 1980s to 1990s, Wojtkowski, Donnay, Markarian and Bunimovich extended the defocusing mechanism to more billiards including Wojtkowski billiards with the Wojtkowski condition. Wojtkowski condition requires long free path length for parallel beams after focusing wall collision trajectories start to focus on some point and then afterwards become dispersing before arriving next non-flat wall collision.
Lemon billiard is a type of billiards which strongly violates the Wojtkowski condition. In 2013, Bunimovich, Zhang, Zhang pioneered the study of Lemon Billiards' hyperbolicity. In 2021, Jin, Zhang proved a condition for Lemon billiards to have hyperbolicity for a section return map thus for the collision map itself.
This poster is to discuss a result for the Lemon billiards section return maps' uniform expansion/hyperbolicity which will be used later to prove local ergodicity.

06. Vincent Goverse: *Synchronization on average for random circle endomorphisms with zero Lyapunov exponent*

In this poster, we present novel results on random systems of circle endomorphisms. Specifically, we examine the case where the Lyapunov exponent is zero and, under some mild assumptions, demonstrate synchronization on average, though not synchronization. Additionally, we discuss how this behavior can be deduced from the existence of sigma-finite stationary measure for the two-point motion. This is a joint work with M. Engel, A.J. Homburg, D. Turaev and J.S.W. Lamb

07. Dániel Jánosi: *Describing chaos in non-autonomous Hamiltonian systems*

When studying chaotic non-autonomous systems, the traditional "one-trajectory" method becomes unreliable, since the dynamics changes in every instant. This has been well understood in climate dynamics, where numerous studies in recent decades have been focusing on analysing an ensemble of trajectories (parallel climate realizations), leading to the construction of the so-called snapshot attractor of the climate. In the theory of low-dimensional non-autonomous chaotic dissipative systems, this refers to a time-dependent object in phase space which all trajectories converge to; a concept that has been well worked out and understood over the decades. However, the same cannot be said about Hamiltonian systems of the same kind, about which virtually no literature existed before, despite such systems having a number of useful applications. In my poster, I present some of the concepts we developed over the past years to describe chaos in these systems. One of the most important of these concepts is that of the snapshot torus. This is the generalisation of KAM tori of autonomous Hamiltonian systems, its shape being time-dependent, and its dynamics having a rate-induced transition from regular to chaotic through a break-up process. In the same manner, we can generalise chaotic seas, obtaining snapshot chaotic seas of time-dependent shape. When investigating the dynamical instability, we introduced an instantaneous Lyapunov exponent, which can be obtained through a quantity called ensemble-averaged pairwise distance (EAPD). Besides these, we generalised a number of other important concepts, such as elliptic and hyperbolic fixed points or stable and unstable manifolds.

08. Niels Kolenbrander: *On some random billiards in a tube with superdiffusion*

We consider a class of random billiards in a tube, where reflection angles at collisions with the boundary of the tube are random variables rather than deterministic (and elastic) quantities. We obtain a (non-standard) Central Limit Theorem for the horizontal displacement of a particle, which marginally fails to have a second moment w.r.t. the invariant measure of the random billiard.
Joint work with: Dalia Terhesiu and Henk Bruin

09. Matheus Manzatto de Castro: *Conditioned Stochastic Stability of Equilibrium States on Maximal Hyperbolic Invariant Sets*

We study conditioned stochastic stability on maximal hyperbolic invariant sets. We show that quasi-ergodic measures of absorbing Markov processes, generated by random perturbations of the deterministic dynamics and conditioned upon survival in a neighborhood of such regions, converge to an invariant measure in the zero-noise limit. This result extends previous findings on uniformly expanding repellers. Under suitable choices of random perturbation, we find that equilibrium states on maximal hyperbolic invariant sets are conditionally stochastically stable. This is joint work with Bernat Bassols Cornudella and Jeroen Lamb.

10. Renee Oldfield: *Measure theoretic entropy for a class of random expanding Blaschke product cocycles*

Measure-theoretic entropy, a measure-theoretic invariant of a dynamical system, measures the rate of increase in dynamical complexity over time. We consider a class of expanding Blaschke product cocycles, inner functions formed by taking products of Möbius transforms. The measure-theoretic entropy of a class of non-uniformly expanding Blaschke product cocycles acting on the complex unit circle will be explicitly described. We also obtain a computable formula for the average metric entropy for a class of one parameter families of expanding Blaschke product cocycles, extending the work by E. Pujals, L. Robert and M. Shub (Ergod. Th. & Dynam. Sys. (2006), 26, 1931–1937) to a random setting.

11. Gustavo Pessil: *Scaled thermodynamic formalism for the metric mean dimension*

Metric mean dimension is a geometric invariant of dynamical systems (X, d, T) with infinite topological entropy. It quantifies the rate at which the amount of ε-distinguishable orbits goes to infinity as $\varepsilon \to 0$. As in the topological pressure of finite entropy systems, one can add the dependence on a continuous potential $\varphi\colon X \to \mathbb{R}$.

Being a renormalization of the entropy, which now depends on the choice of equivalent metric to generate the topology, it is natural to search for a measure-theoretic notion of metric mean dimension $\mathcal{H}$ satisfying the classical variational principle, namely

$$\mathrm{mdim}_M(X, d, T, \varphi) = \sup_{\mu} \left\{ \mathcal{H}(\mu) + \int \varphi \, d\mu \right\}.$$

We will define such an object, state the variational principle and compute it explicitly for some classical examples of the theory. On these examples the obtained formula for the metric mean dimension with potential will be given in terms of ergodic optimization.

[1] M. Carvalho, G. Pessil and P. Varandas. A convex analysis approach to the metric mean dimension: limits of scaled pressures and variational principles. Adv. Math. 436 (2024) 109407

12. Joshua Peters: *Averaging for random metastable systems*

Random metastability occurs when an externally forced or noisy system possesses more than one state of apparent equilibrium. This work investigates a class of random dynamical systems, arising from perturbing a one-dimensional piecewise smooth expanding map of the interval with two invariant subintervals, each supporting a unique ergodic absolutely continuous invariant measure. Upon perturbation, this invariance is destroyed, allowing trajectories to randomly switch between subintervals. We show that the invariant density of the randomly perturbed system may be approximated by an explicit convex combination of the two initially invariant densities, obtained by averaging. Further, we also identify the limit of the second Oseledets space, or coherent structure, as the perturbation shrinks to zero. Our results are applied to random paired tent maps over ergodic, measure-preserving, and invertible driving systems. Finally, we provide generalisations to systems admitting more than two initially invariant sets.

13. Giuseppe Tenaglia: *Non uniformly expanding random systems with additive noise have dense random horseshoes*

We propose a notion of random horseshoe and prove density of random horseshoes for one dimensional random dynamical systems with additive noise and positive Lyapunov exponent