

FLOER HOMOLOGY & THE ARC-FLOER CONJECTURE

Simons School on Higher Dimensional Varieties.

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PROPERTIES OF $HF(\varphi^m, +)$

(A1) isotopy invariance

(A2) categorification of $\lambda(\varphi^m)$

(A3) spectral sequence

FLOER
THEORY

what can we learn
about the geometry
of a symplectic
object by studying
pseudoholomorphic
curves?

(ω, J, g)

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Arnold
conjecture counting

$HF(X)$

$Fuk(X)$

$\Pi_{\infty} F(X)$

operations

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(ω, J, g)

functoriality from
cobordisms
"field theory"

FLOER
THEORY

Arnold
conjecture counting

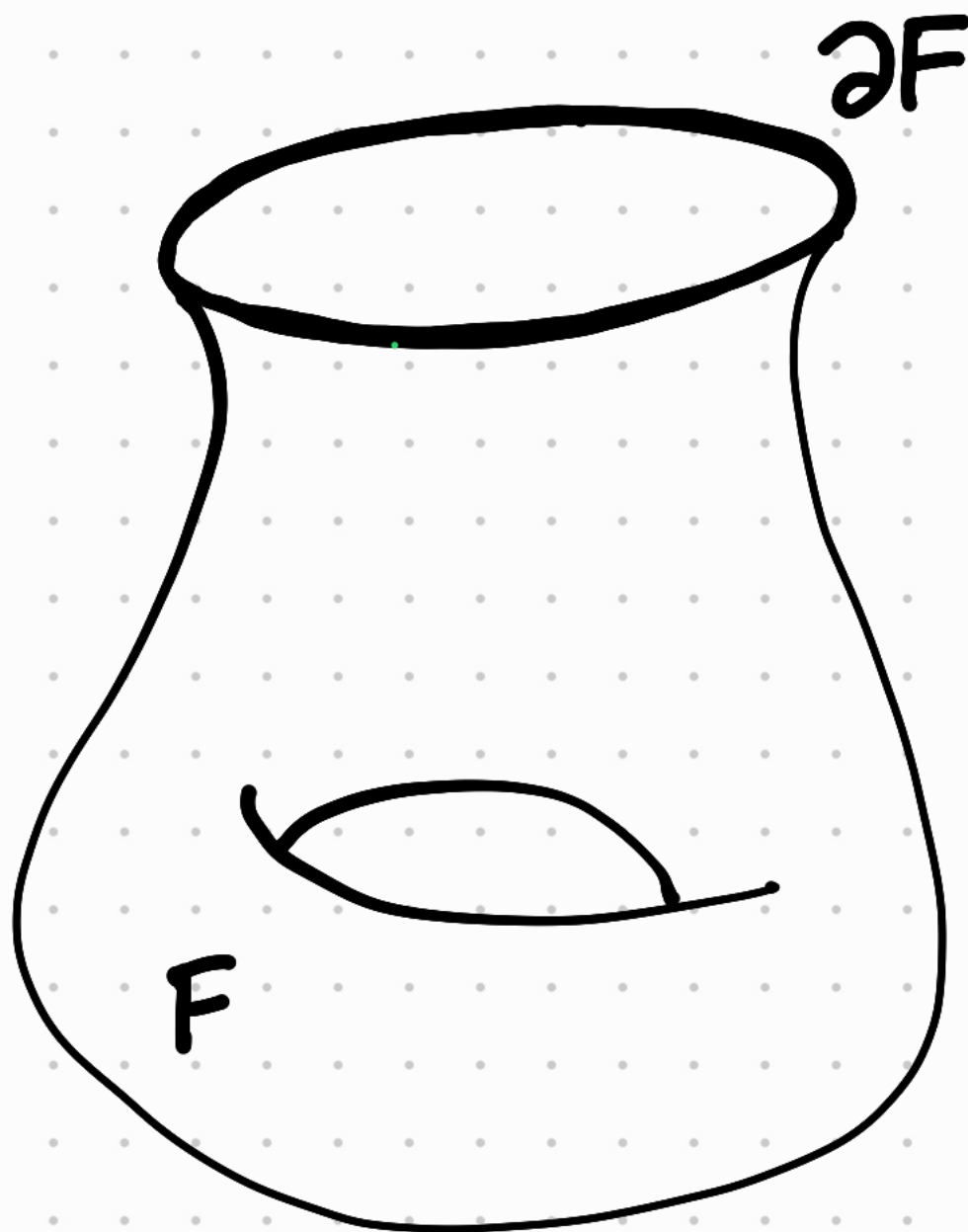
$HF(X)$

$Fuk(X)$

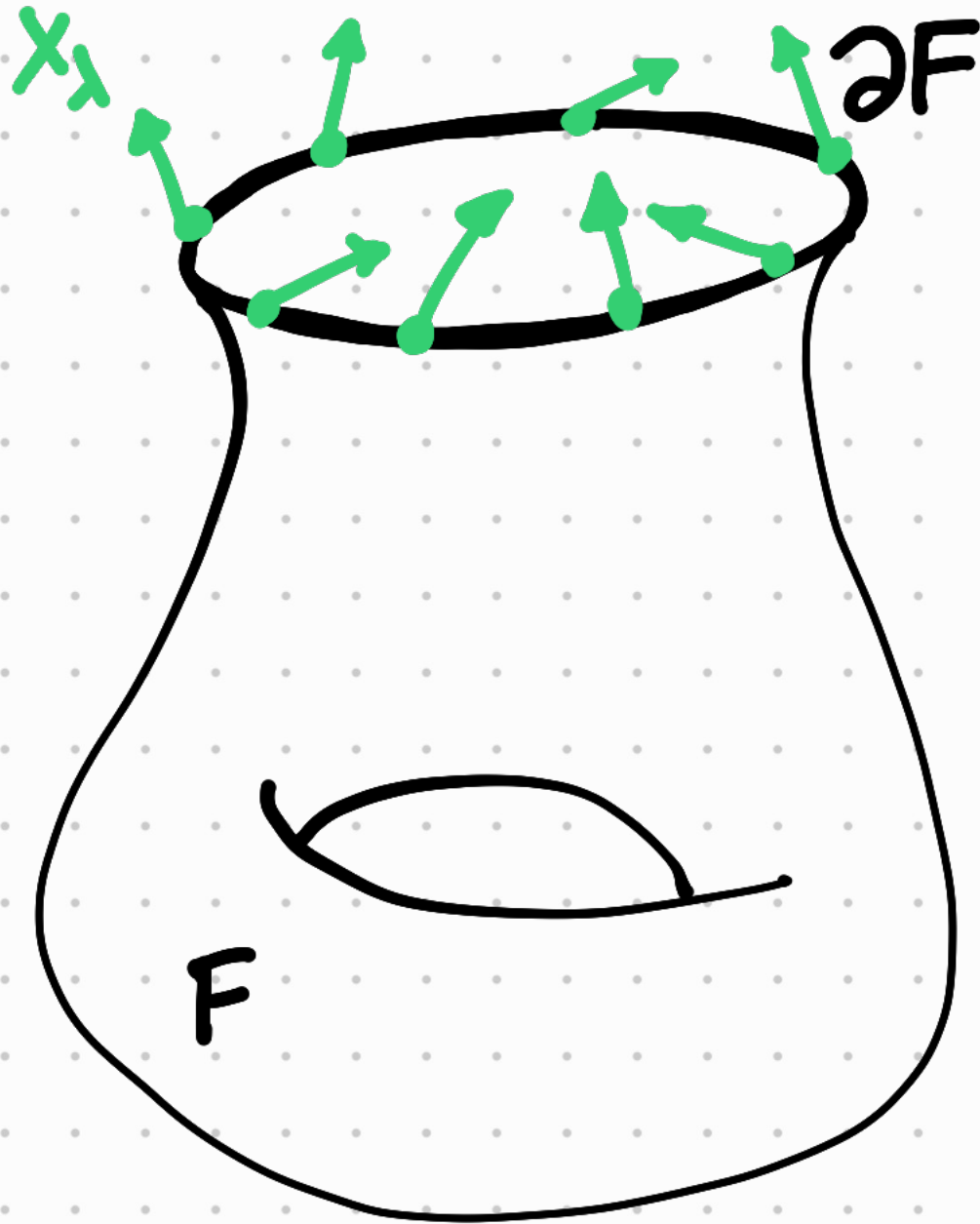
$\Pi_{\infty} F(X)$

operations

how much of the geometry
of X can we recover from
these invariants?

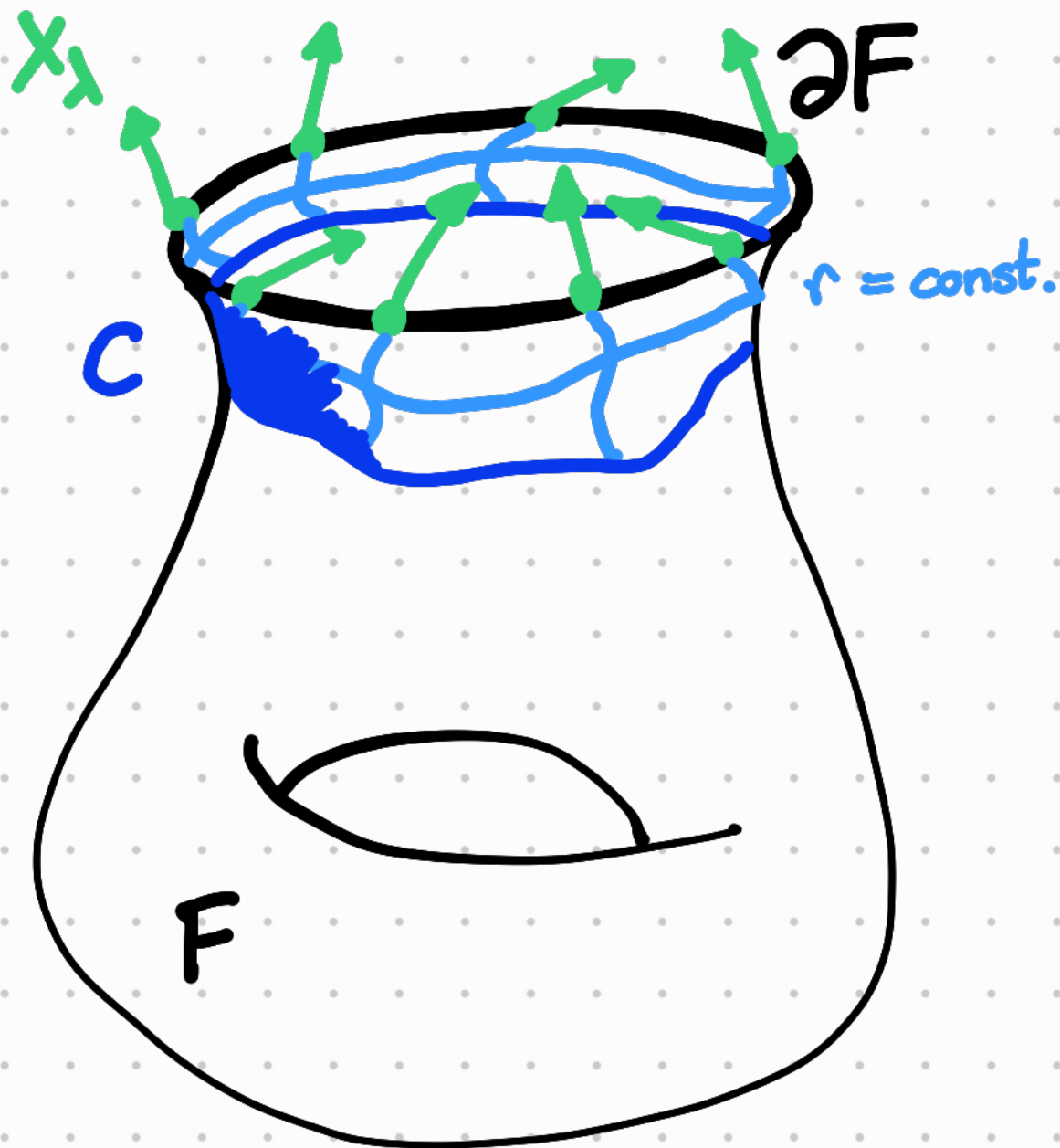


Liouville
vector field



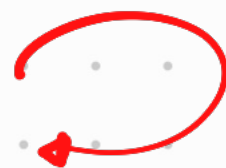
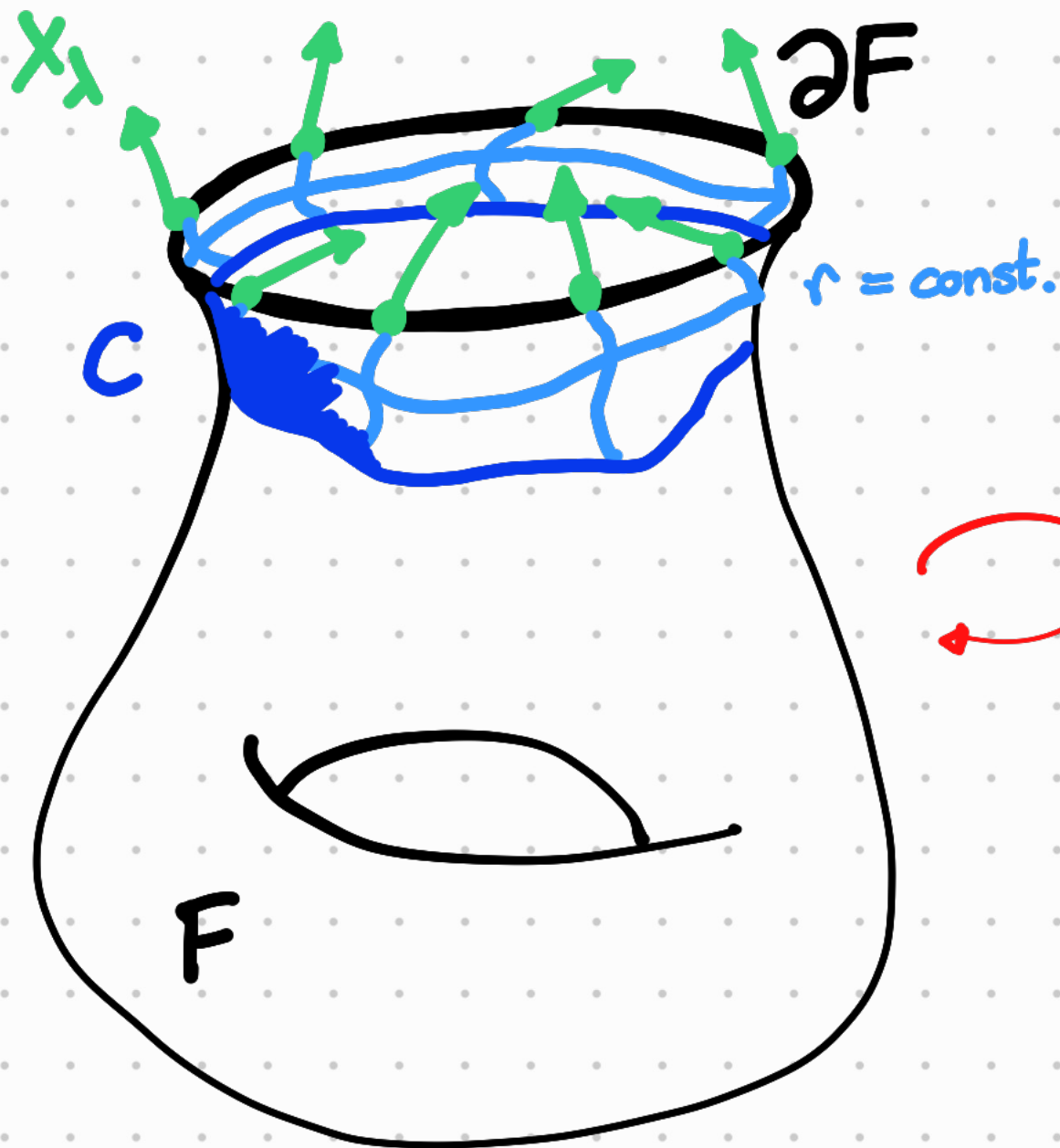
Liouville
vector field

cylindrical
collar

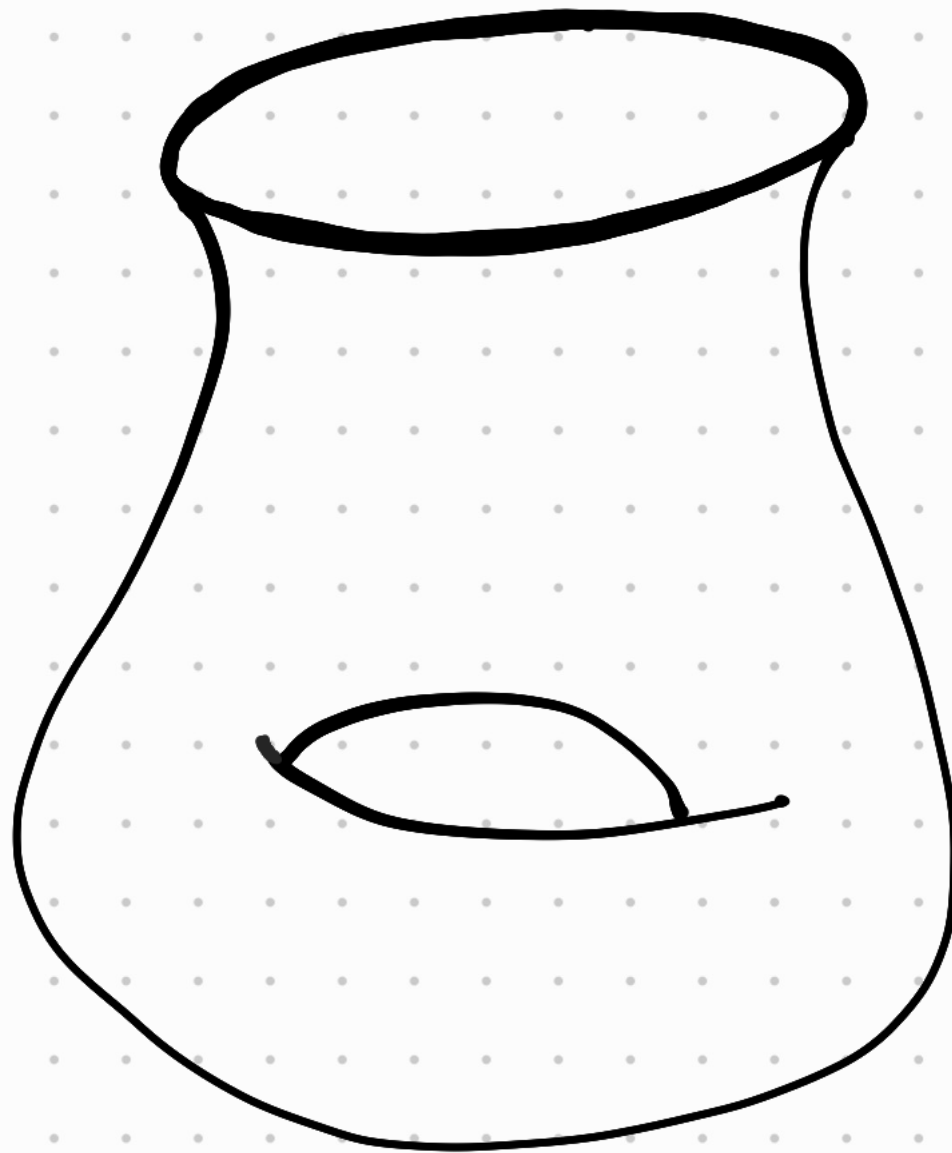


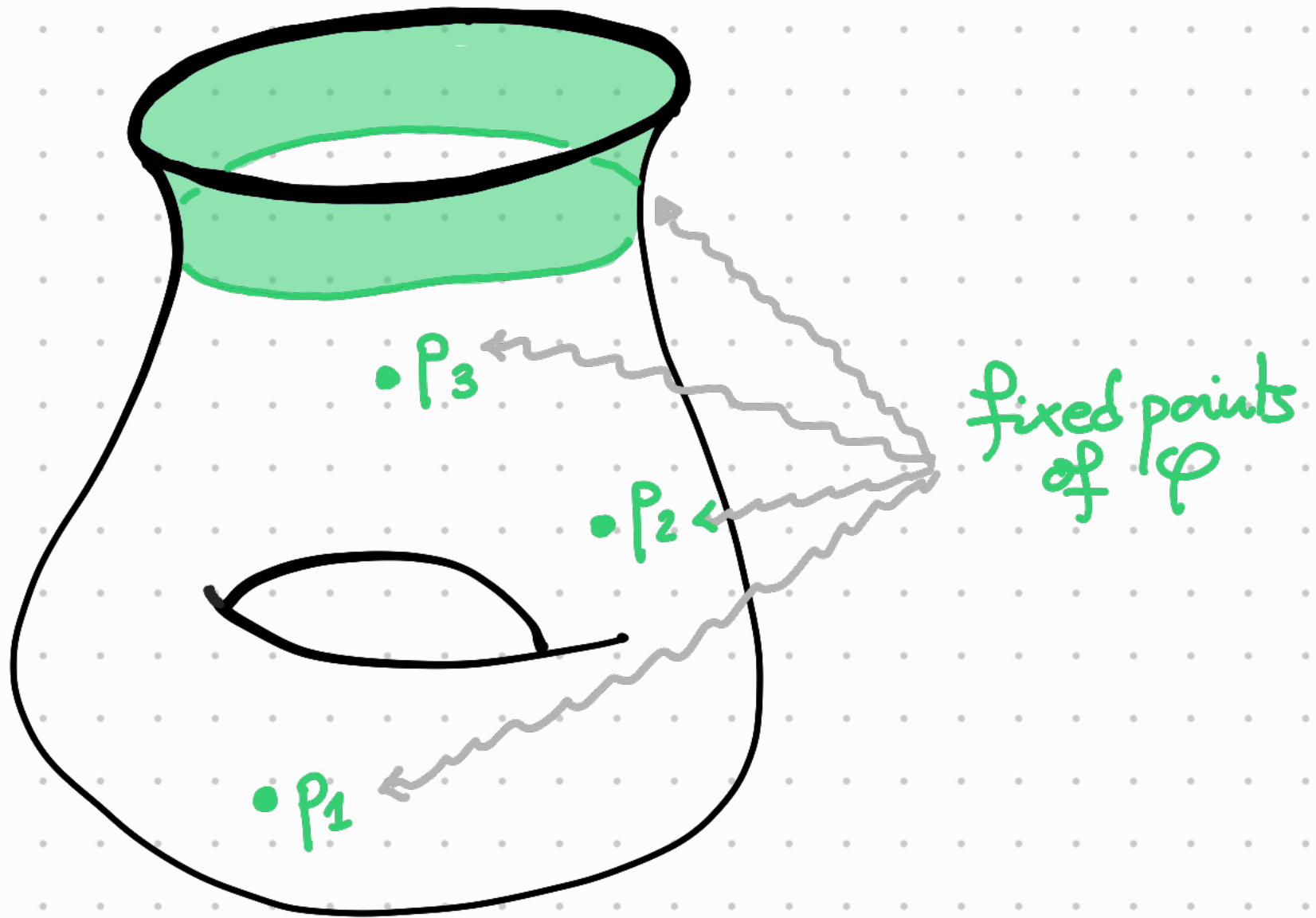
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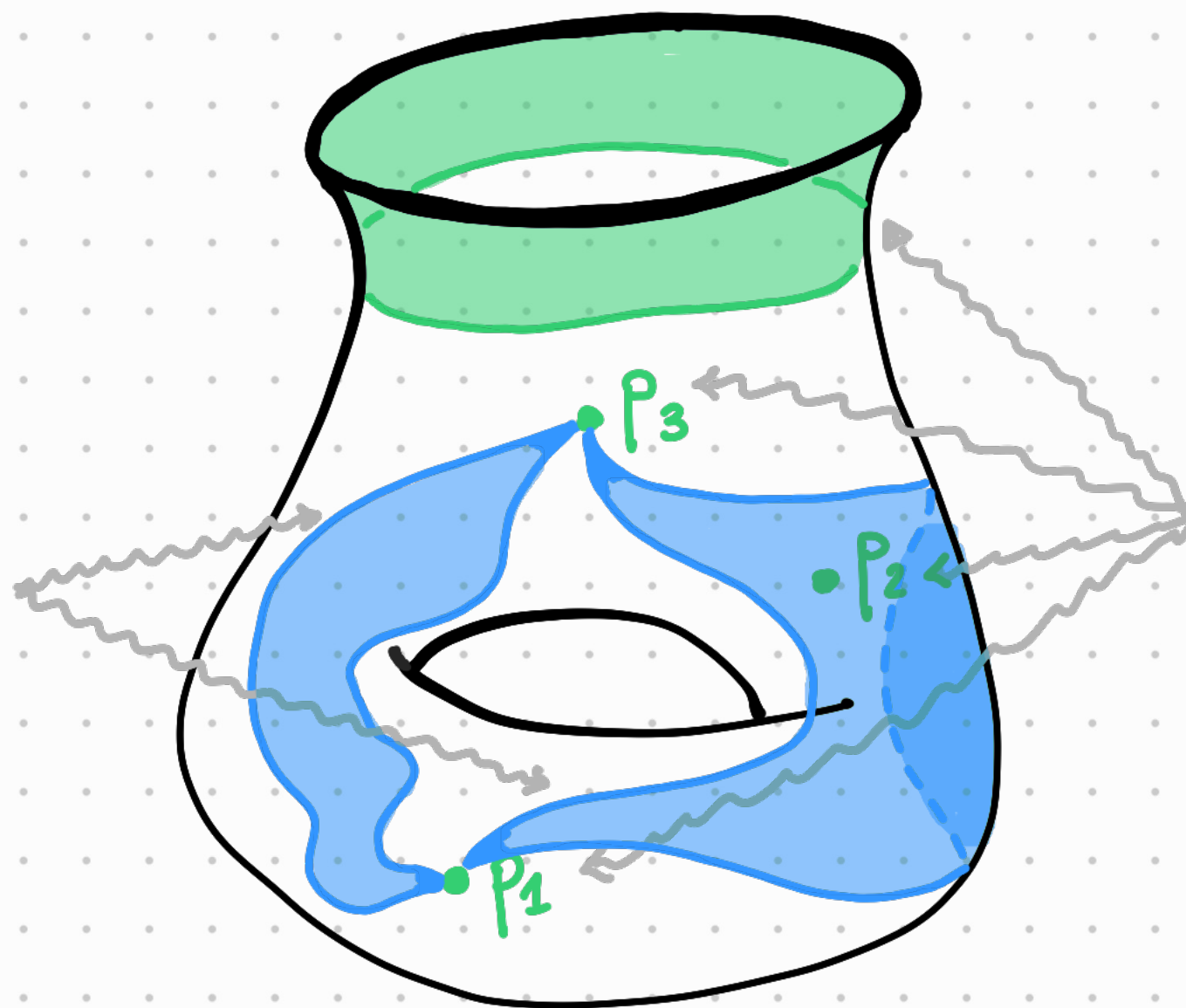


φ compactly
supported
exact
symplectom.



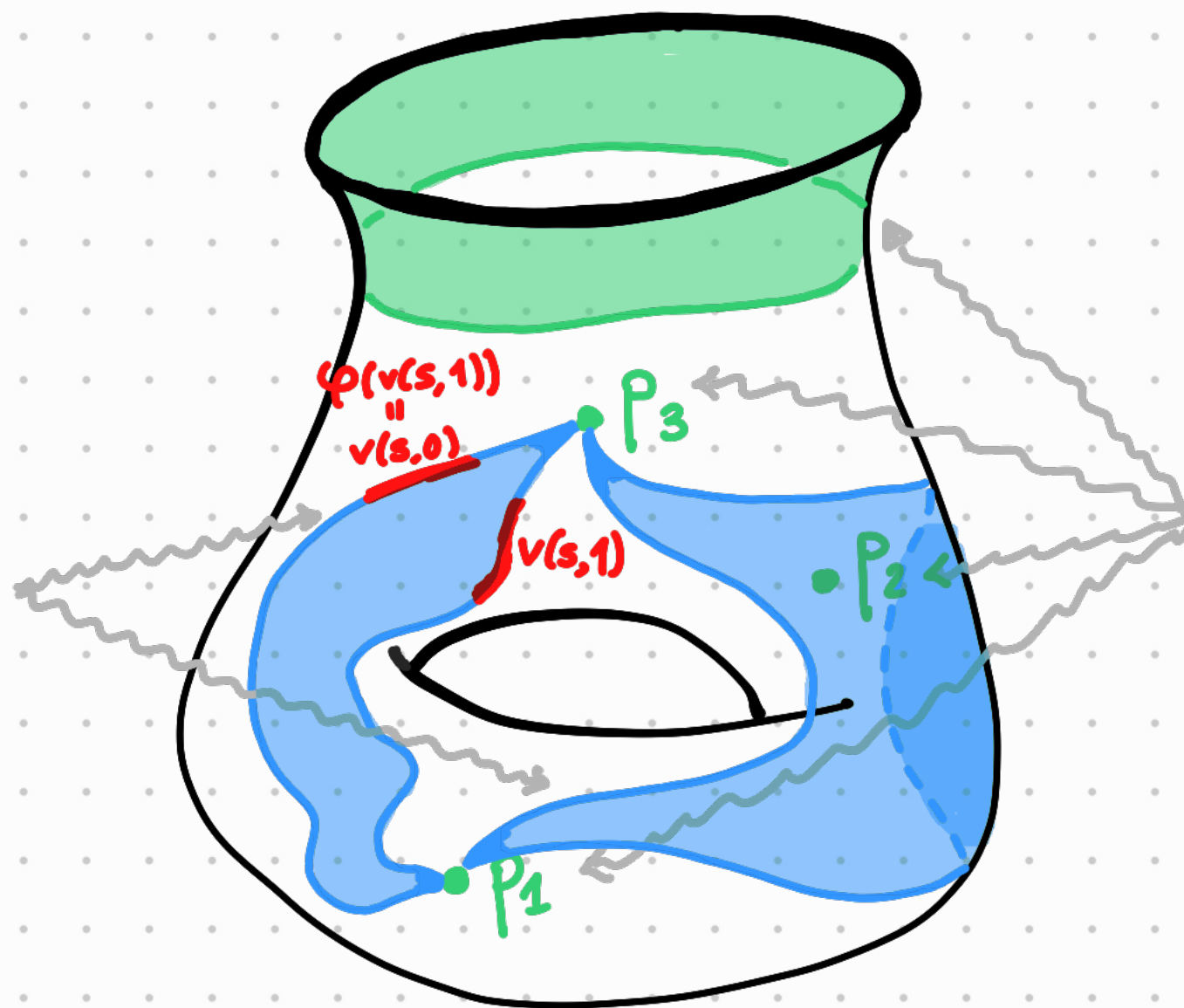


Floer
trajectories

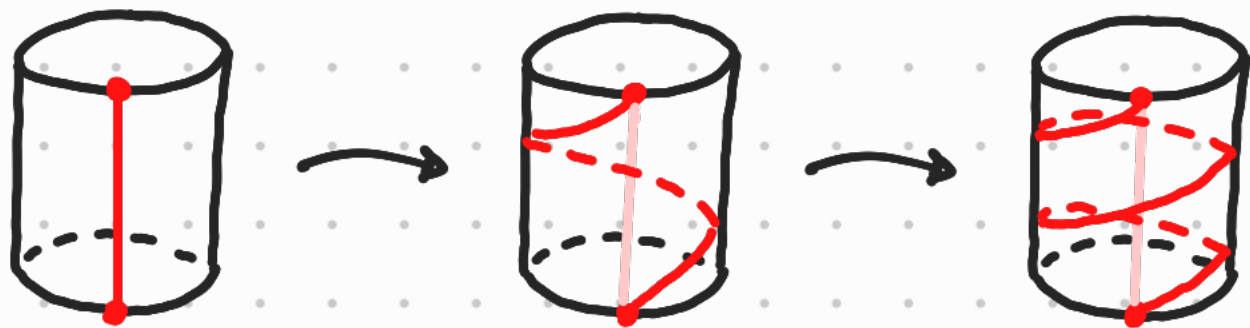


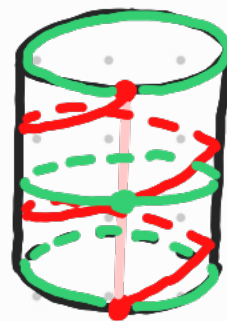
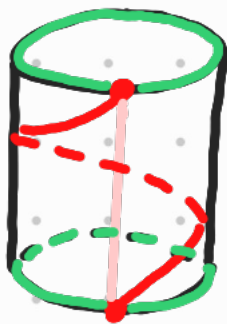
fixed points
of φ

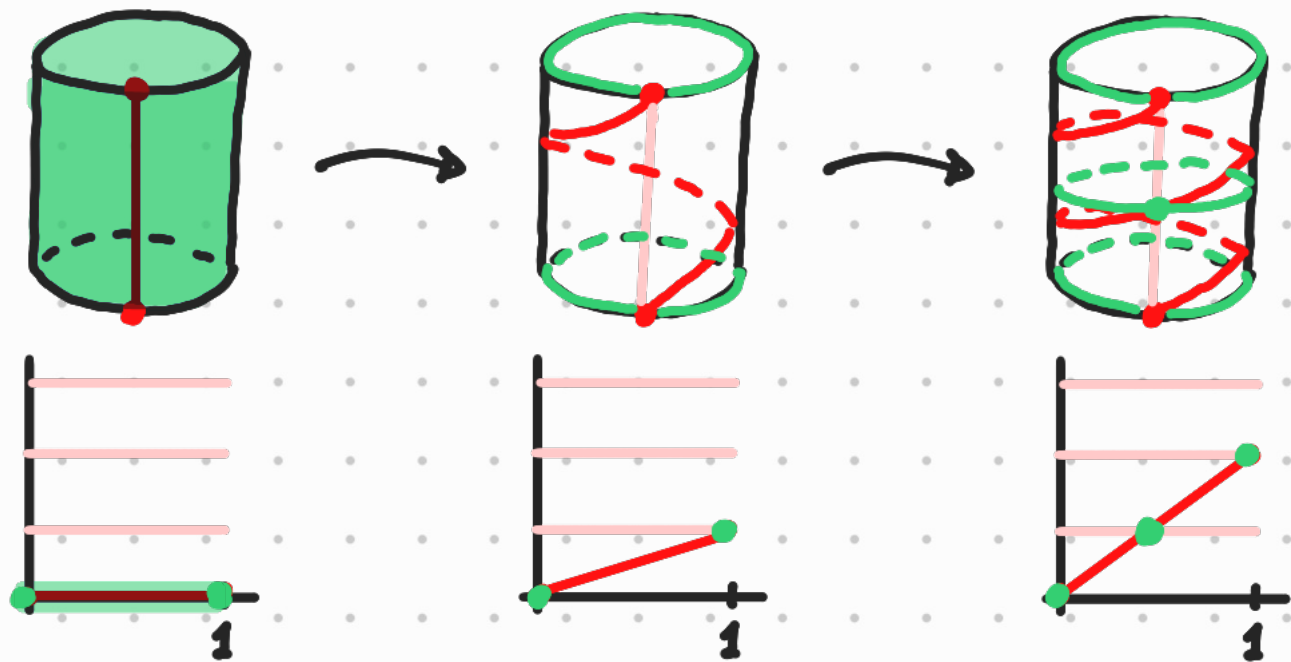
Floer
trajectories

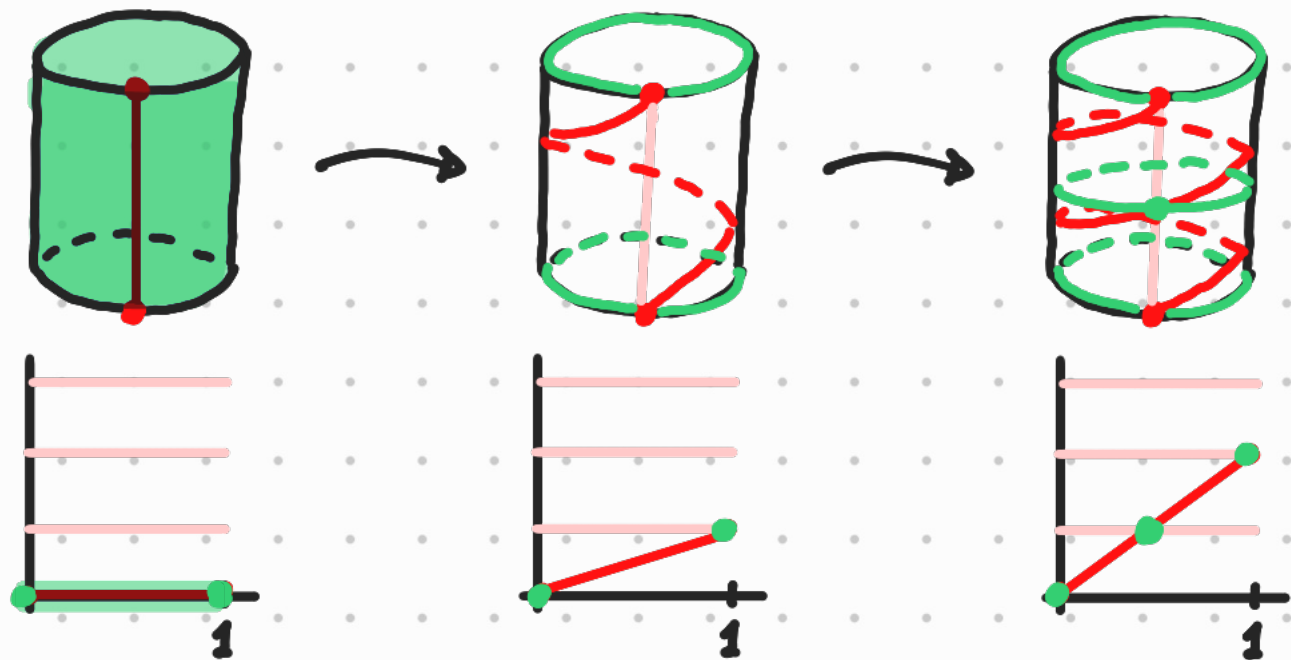


fixed points
of φ

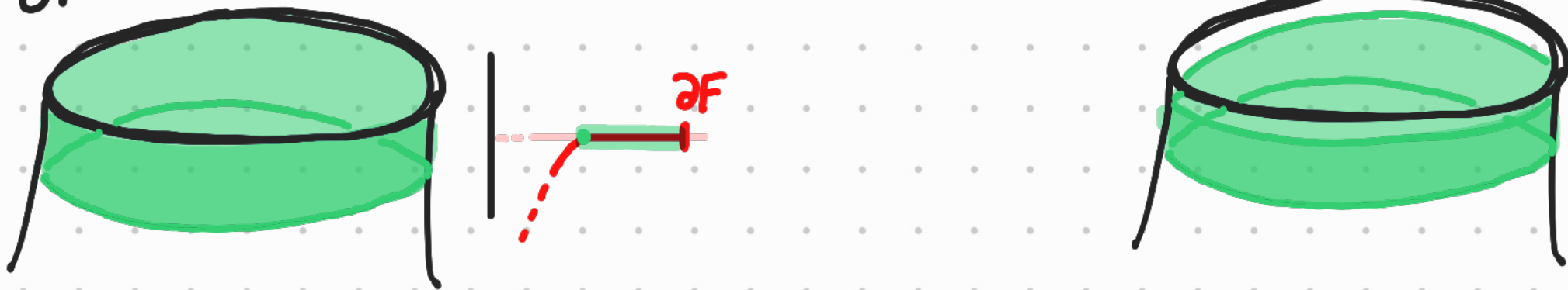


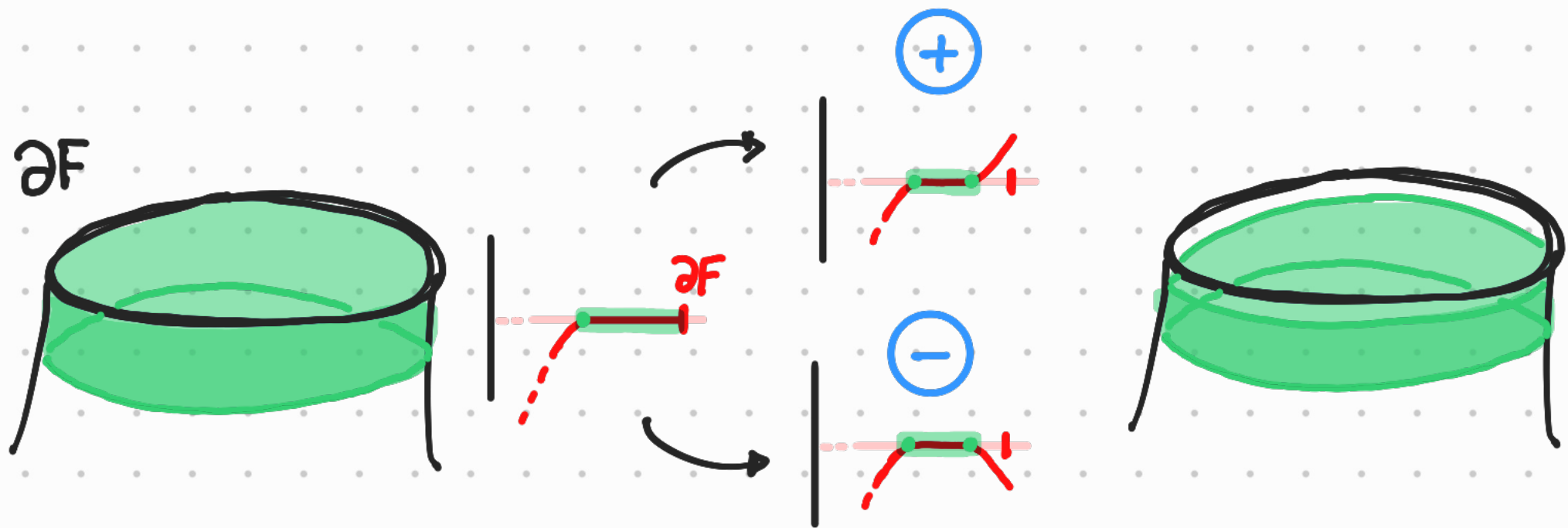
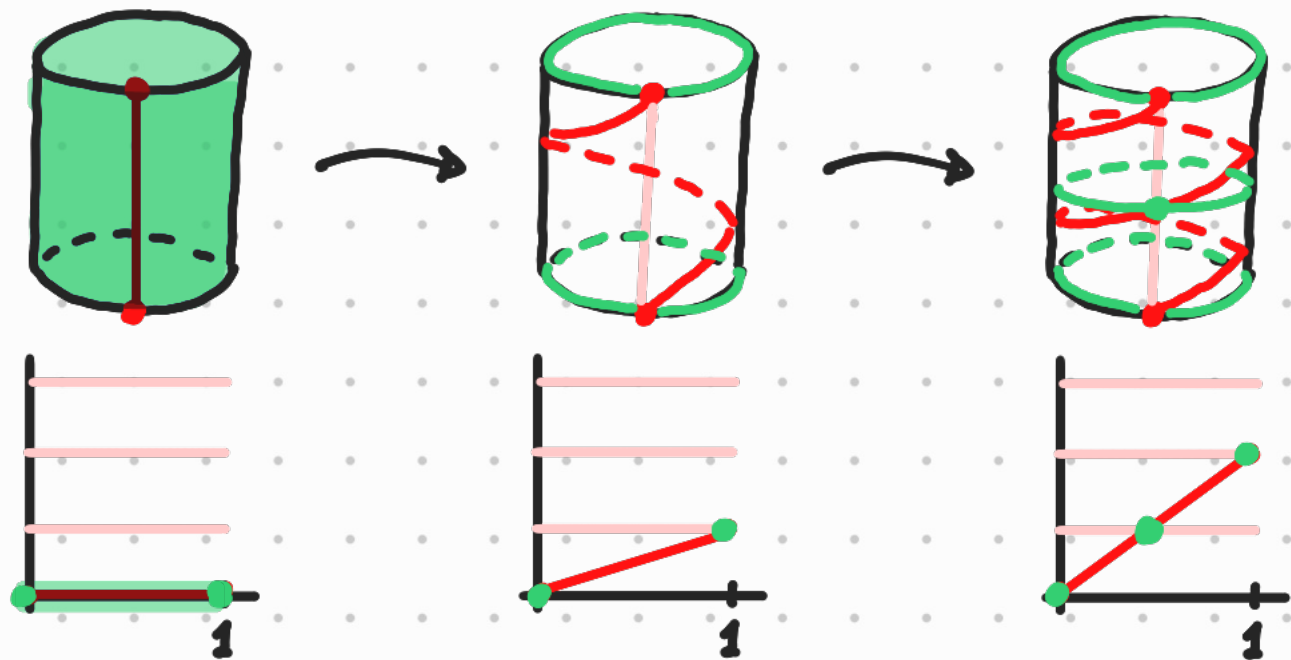




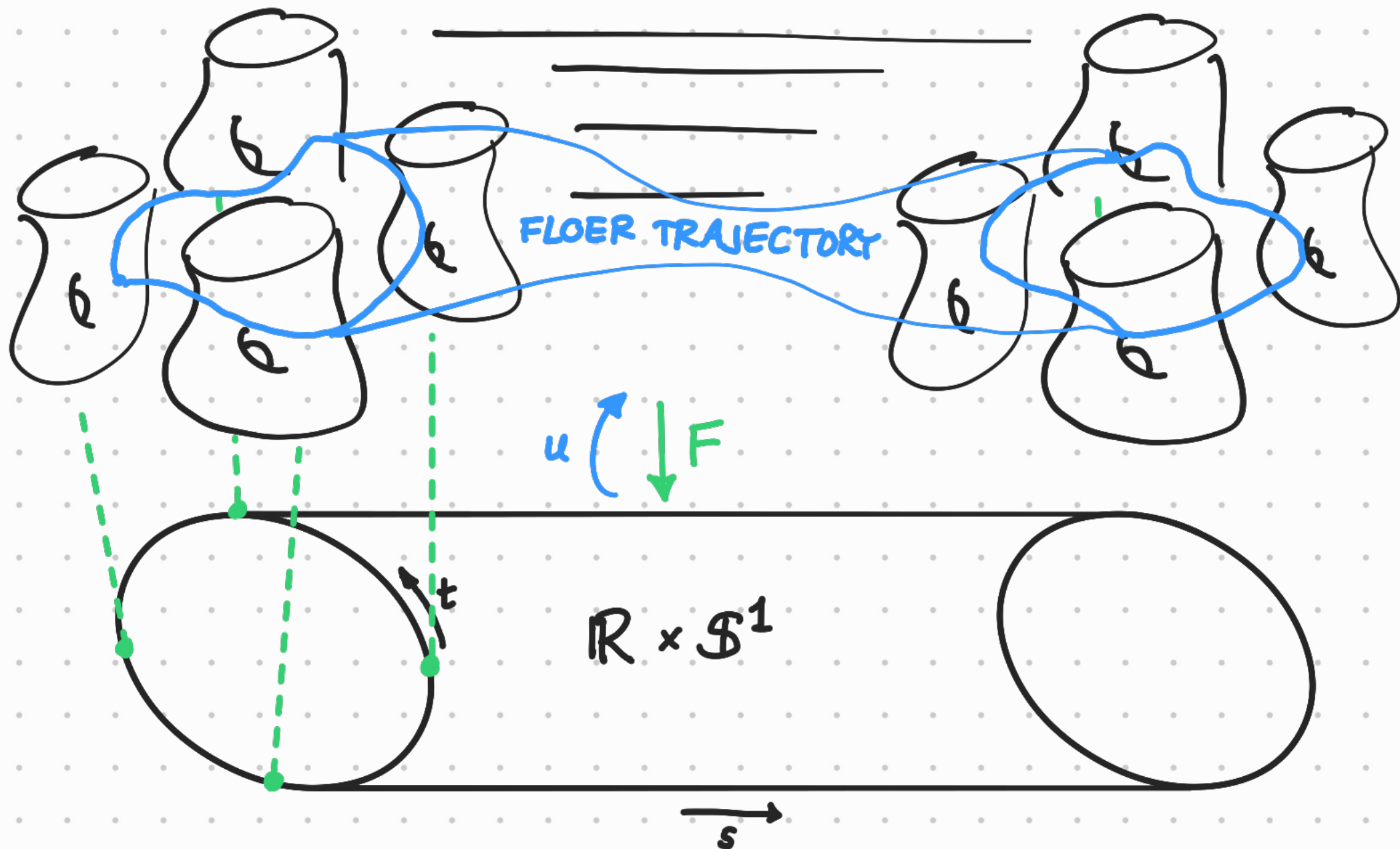


∂F

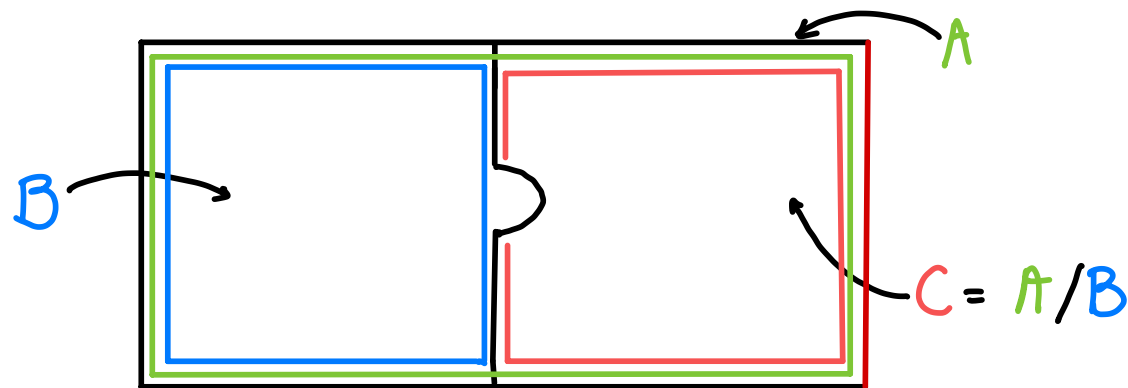




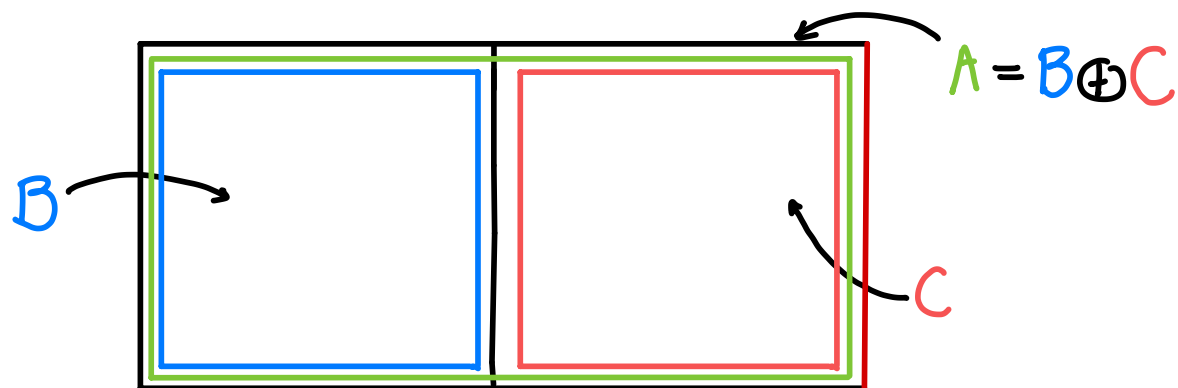
MAPPING TORUS OF φ



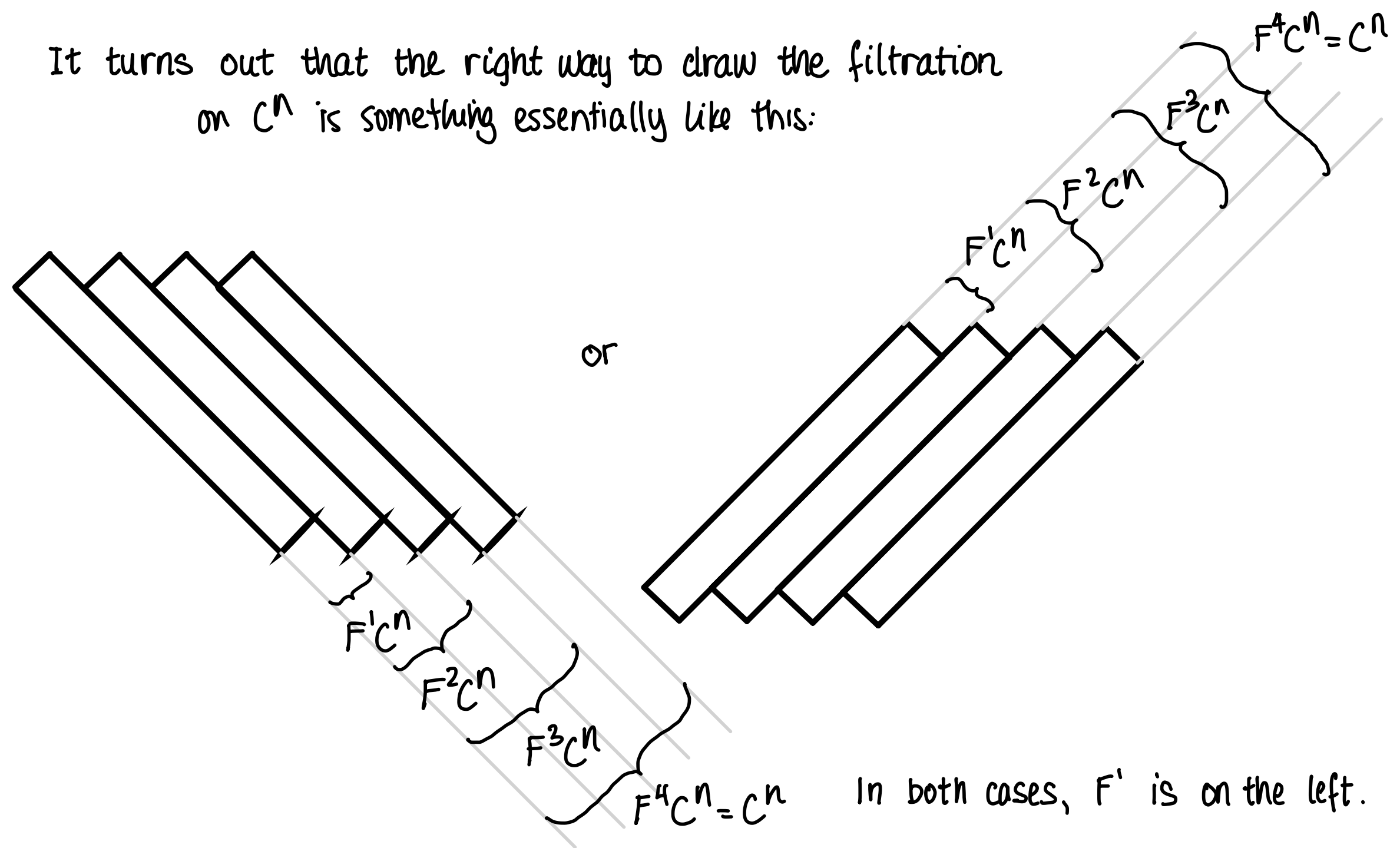
Here is an abelian group A , with a subgroup B , and the corresponding quotient group C .



If $A = B \oplus C$, we might instead draw

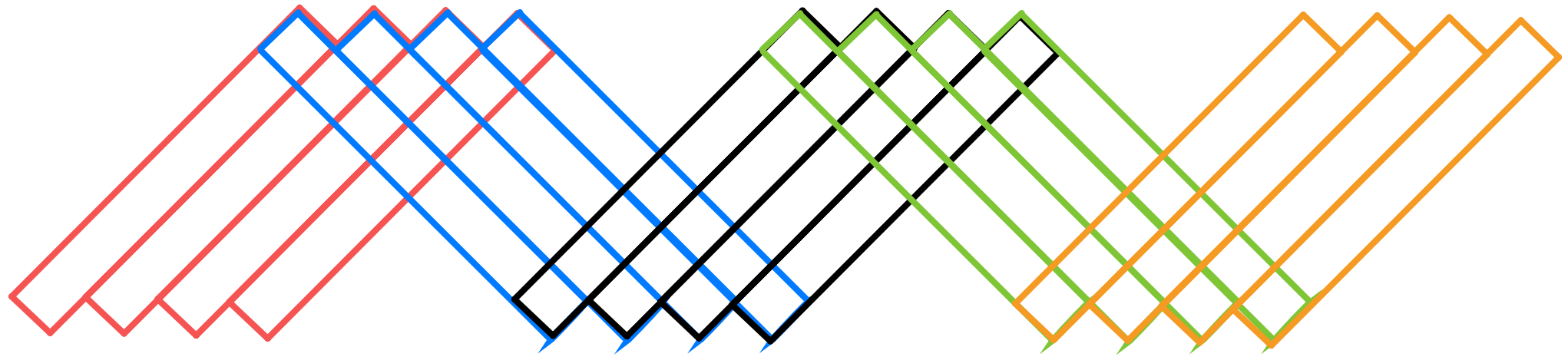


It turns out that the right way to draw the filtration
on C^n is something essentially like this:



In both cases, F^1 is on the left.
In general, "subs" are on the
left and "quotients" are on the right.

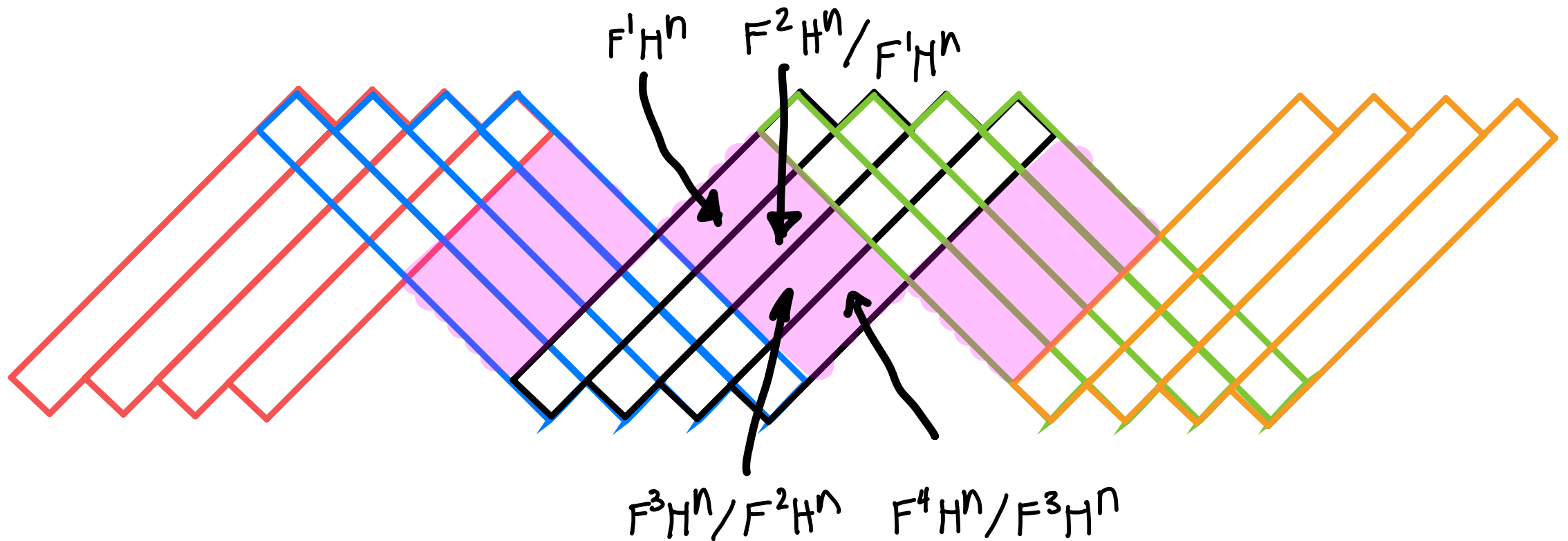
Here then is what the filtered complex looks like.



$$C^{n-2} \xrightarrow{d^{n-2}} C^{n-1} \xrightarrow{d^{n-1}} C^n \xrightarrow{d^n} C^{n+1} \xrightarrow{d^{n+1}} C^{n+2}$$

Here are three cohomology groups, highlighted, corresponding to the part (subquotient) of C^n not overlapping with C^{n-1} or C^{n+1} .

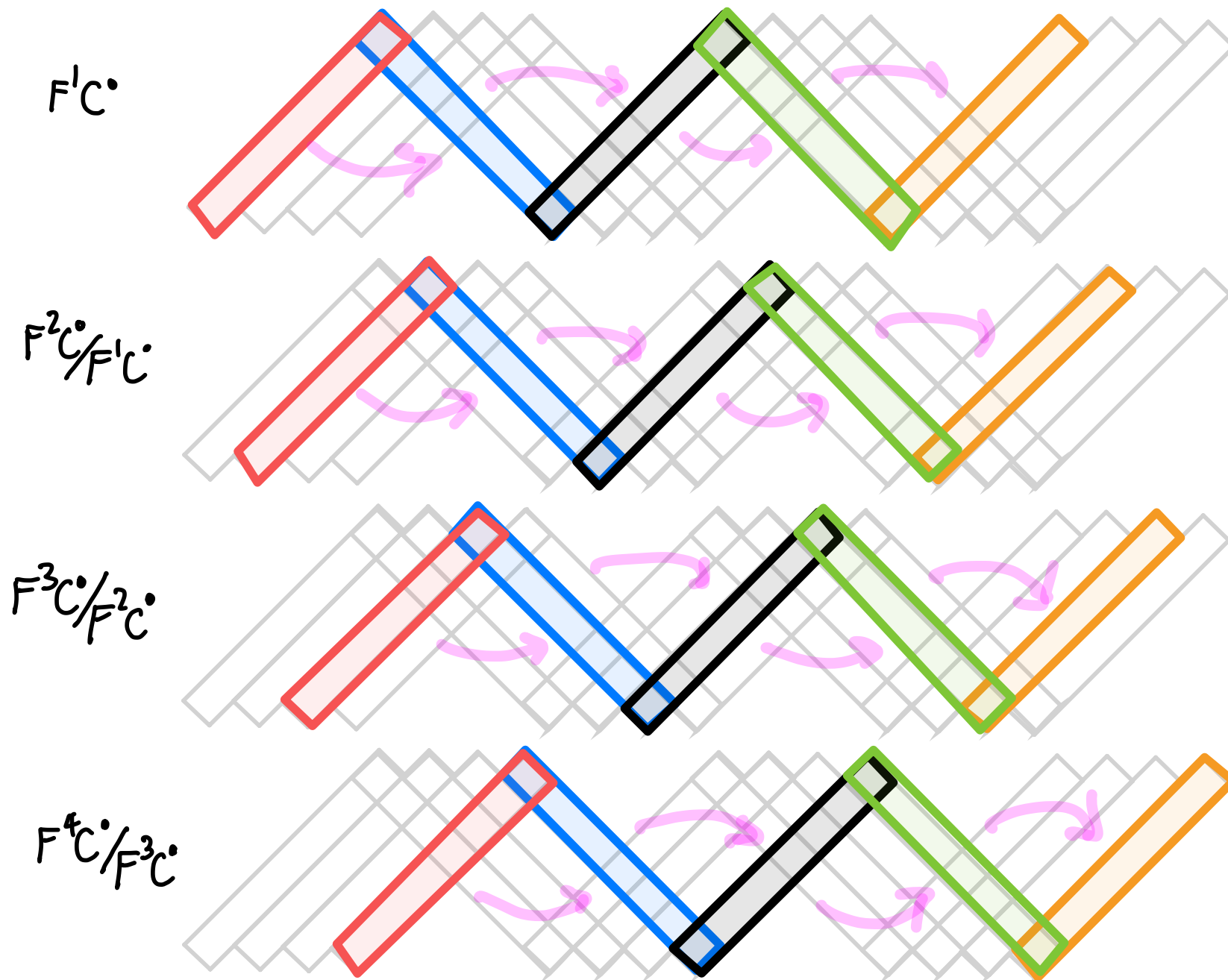
They come with an obvious filtration.



The "output" of the spectral sequence process will be the "parts" of the filtration $F^j H^n / F^{j-1} H^n$.

We begin the process with the "subquotient" complexes, $F^j C^\bullet / F^{j-1} C^\bullet$.

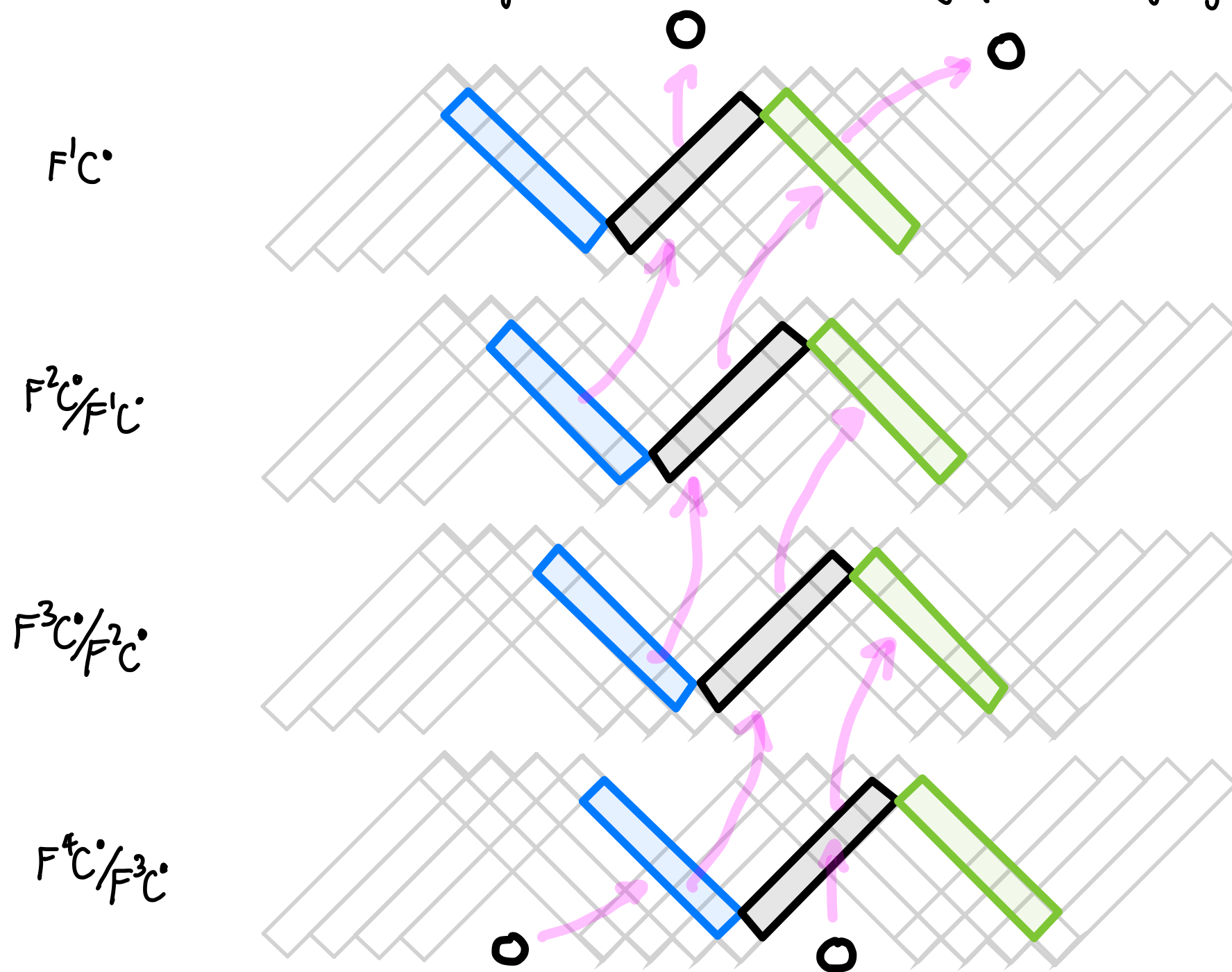
This is "page 0" of the spectral sequence (E_0). Here they are, pictorally.



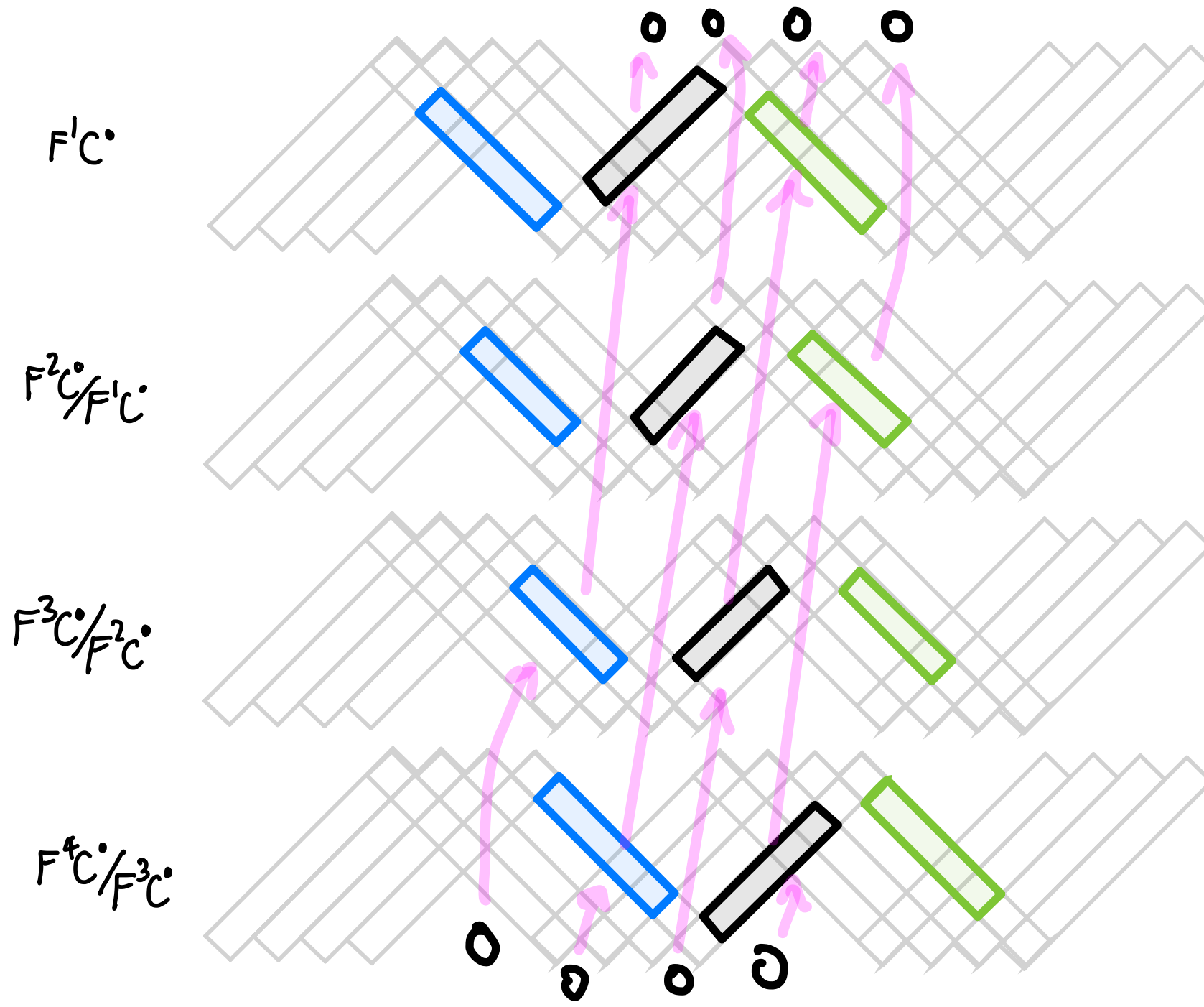
The obvious maps between the spaces (indicated by arrows) clearly describe a series of complexes. We take the cohomology of these complexes to obtain the first page of the spectral sequence, E_1 .

Here is the first page E_1 , with the entries displayed in a similar way.

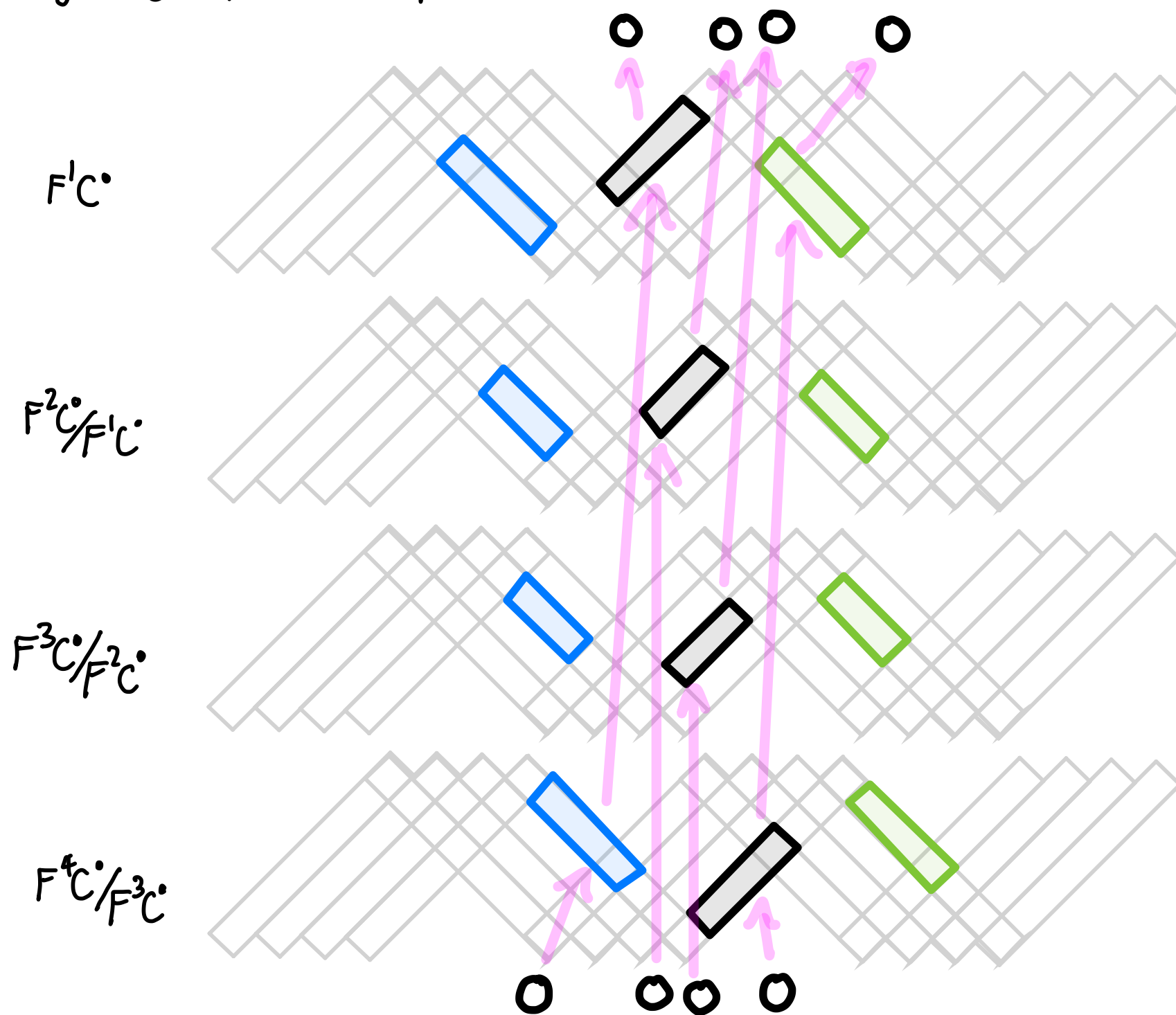
We now have new natural "diagonal" morphisms, clearly (pictorially) yielding complexes.



We now take cohomology of these complexes, obtaining the second page E_2 of the sequence. Now new arrows have revealed themselves.

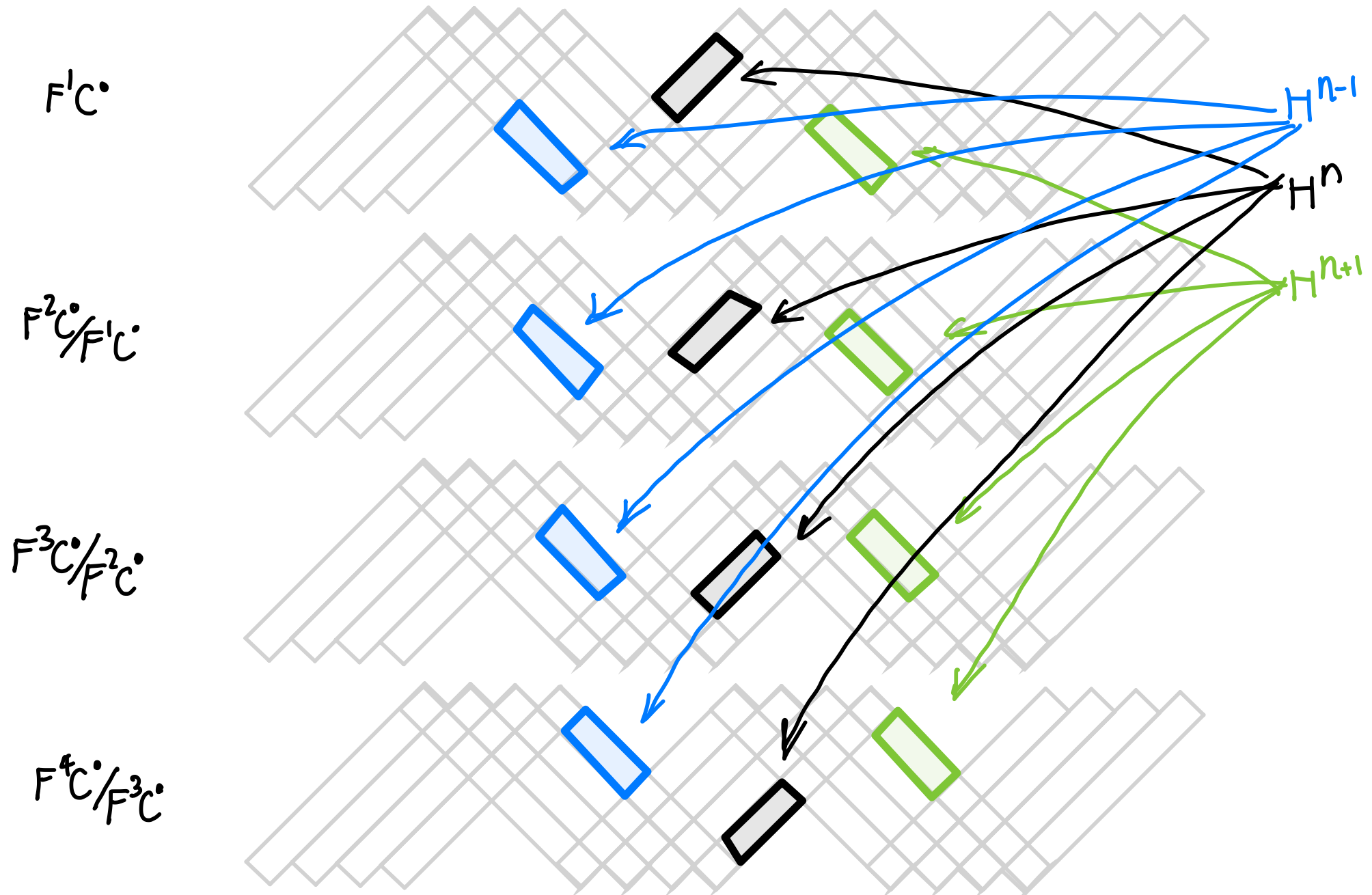


We now take cohomology of these complexes, obtaining the third page E_3 of the sequence. Now new arrows have revealed themselves.



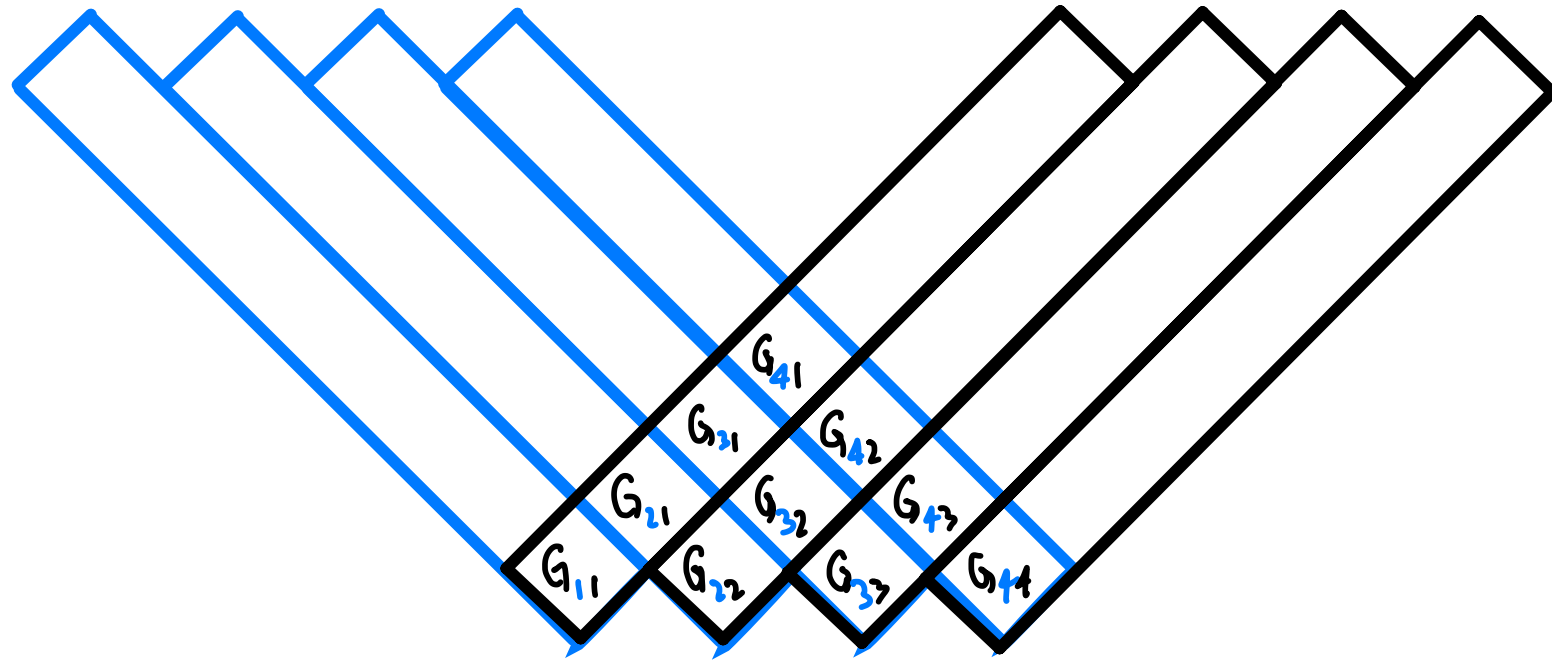
Two of our
four pieces of
 $H^i(C^\bullet)$ have
already
converged

We now take cohomology of these complexes, obtaining the final page E_4 . We have found the desired pieces of $H^*(C^*)$!



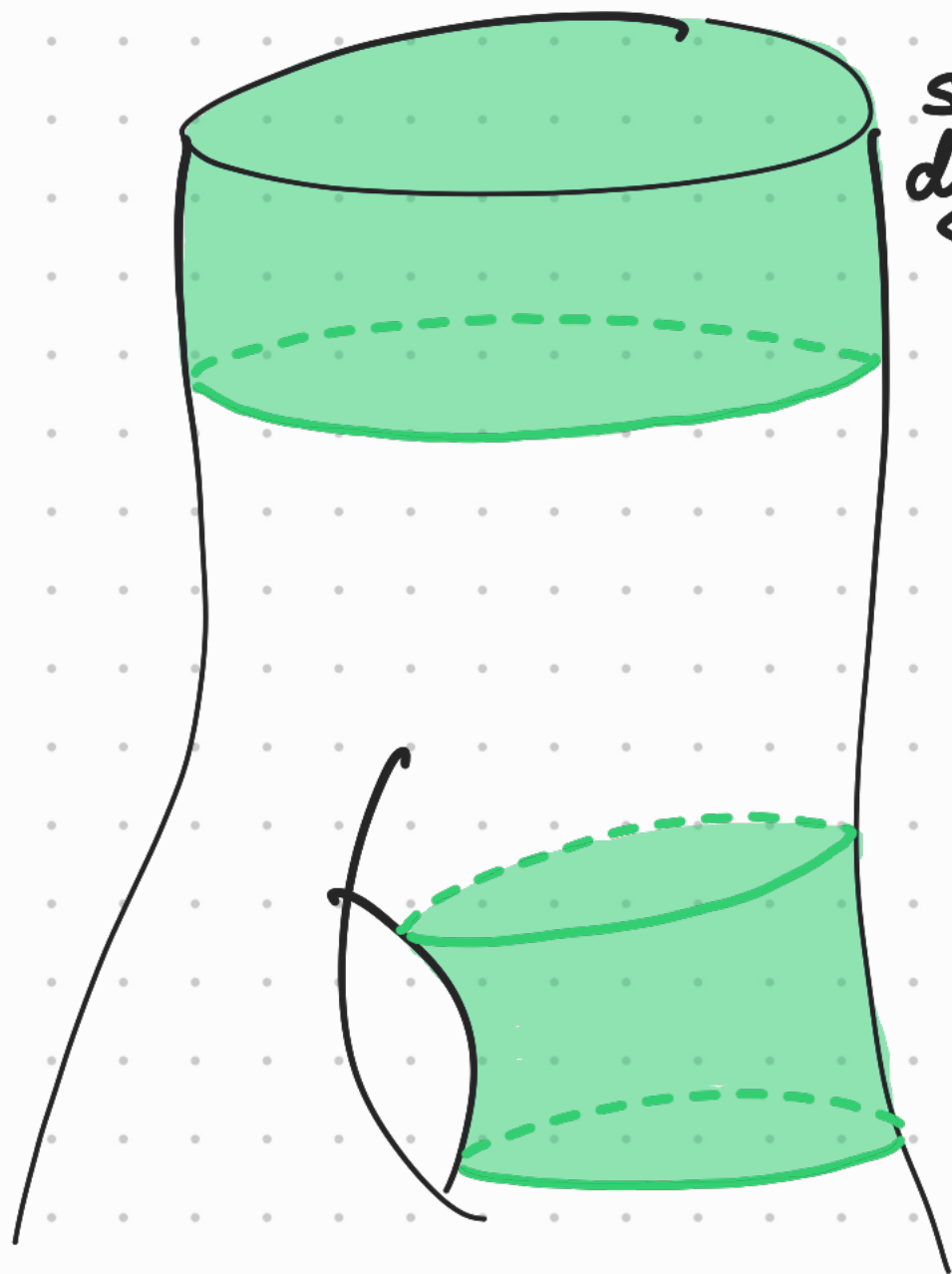
If we look closely, we can see what made this procedure work.

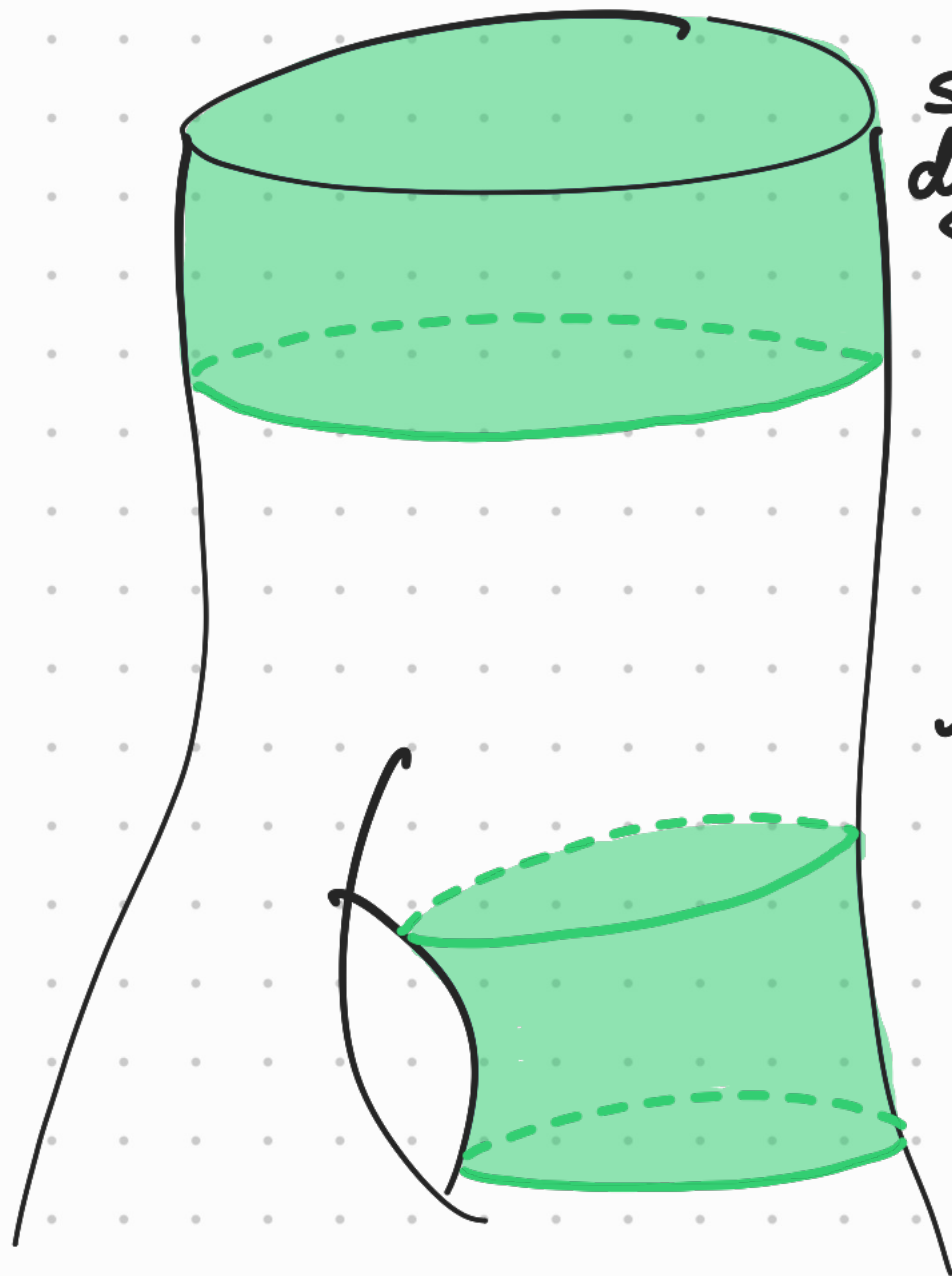
We label some of the regions as follows.



By the p^{th} page E_p , we have surgically removed all the pieces G_{ij} with $i < j+p$.

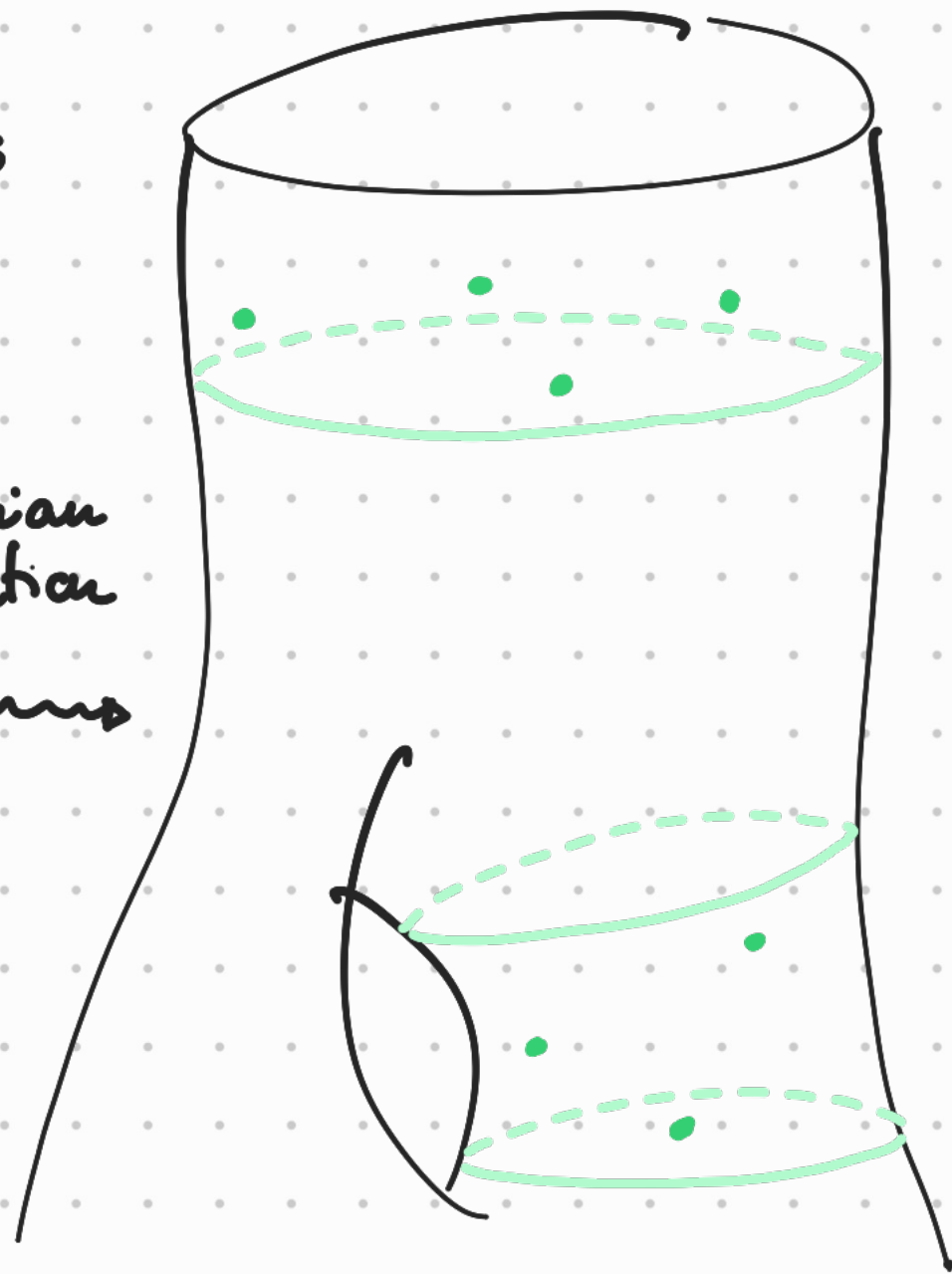
special
dynamics

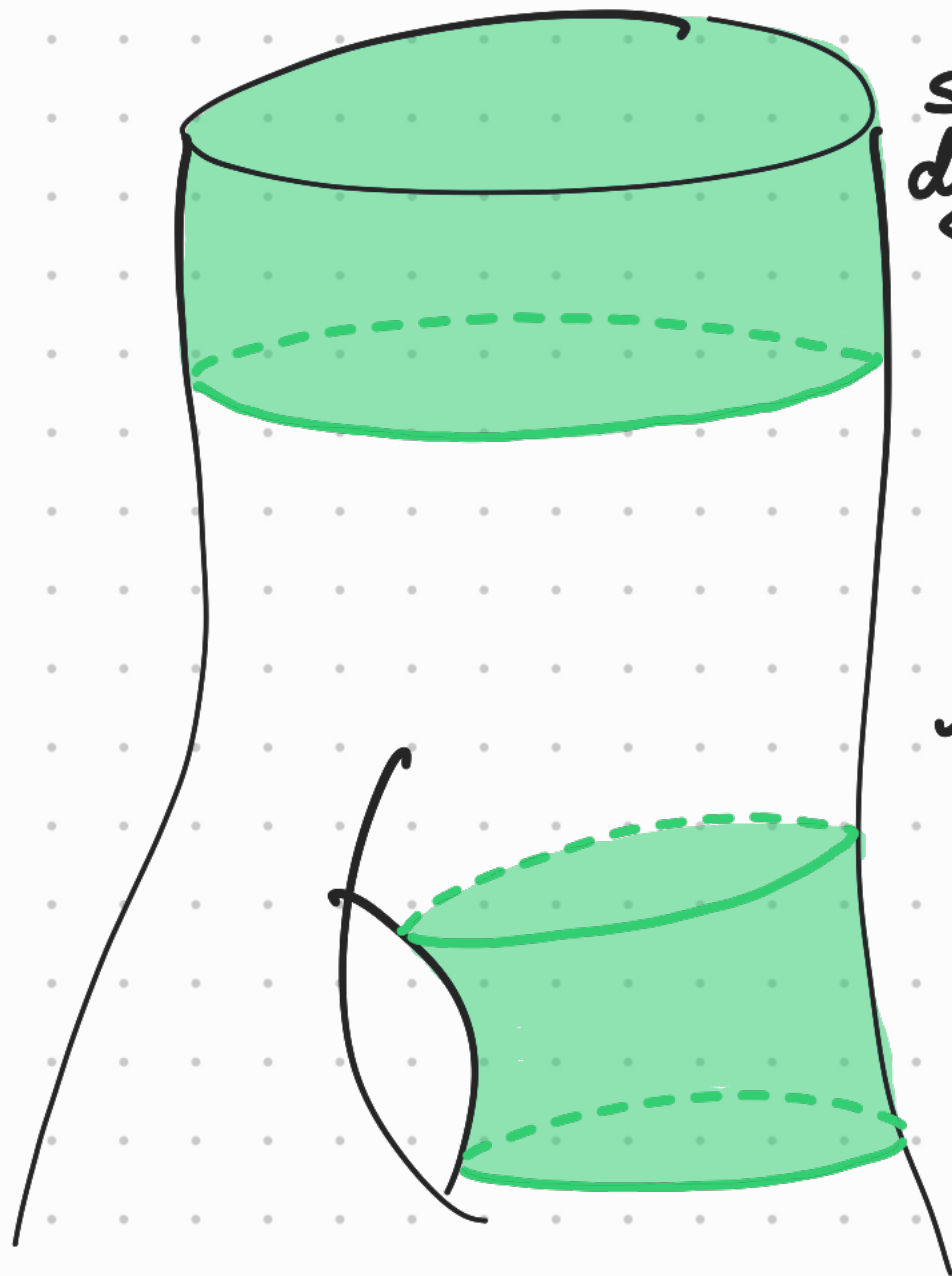




special
dynamics

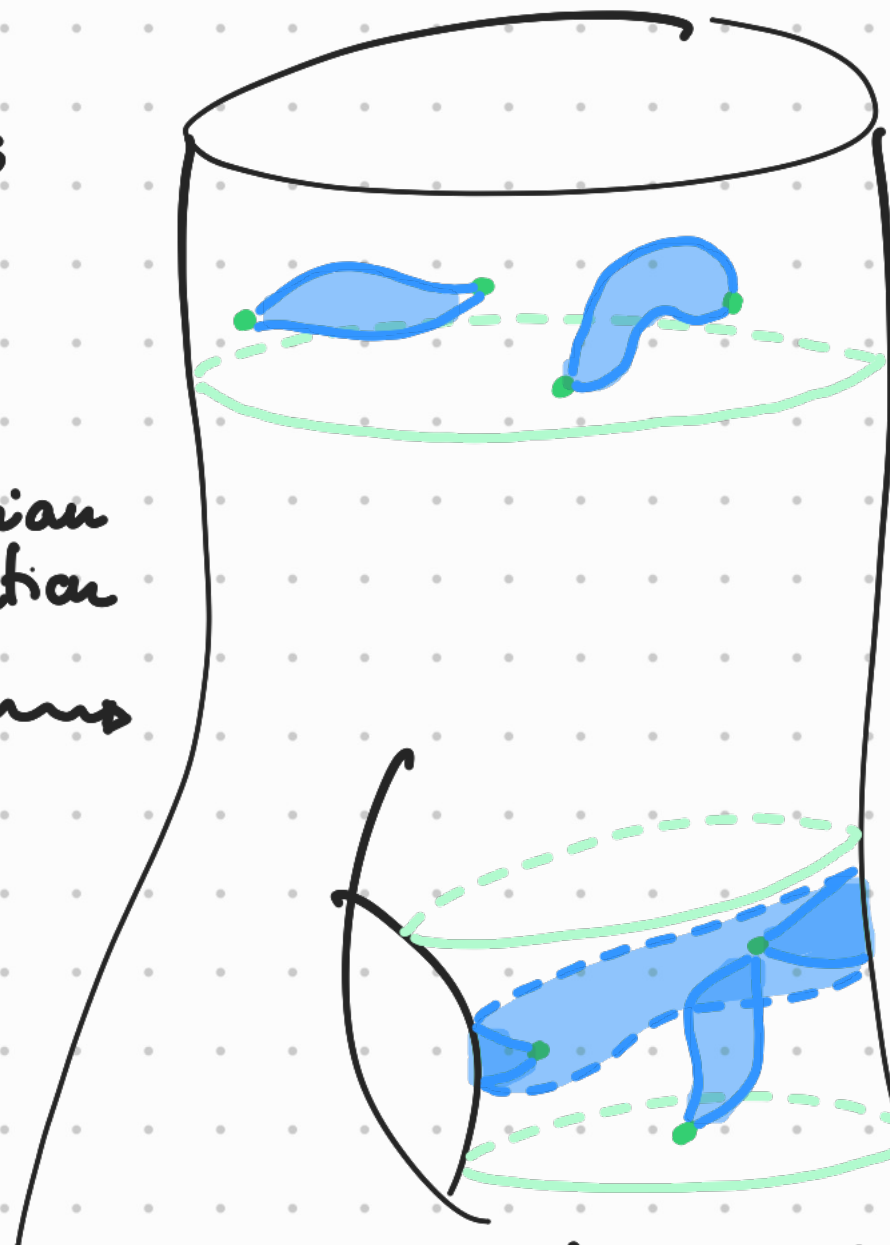
Hamiltonian
perturbation
~~~~~>





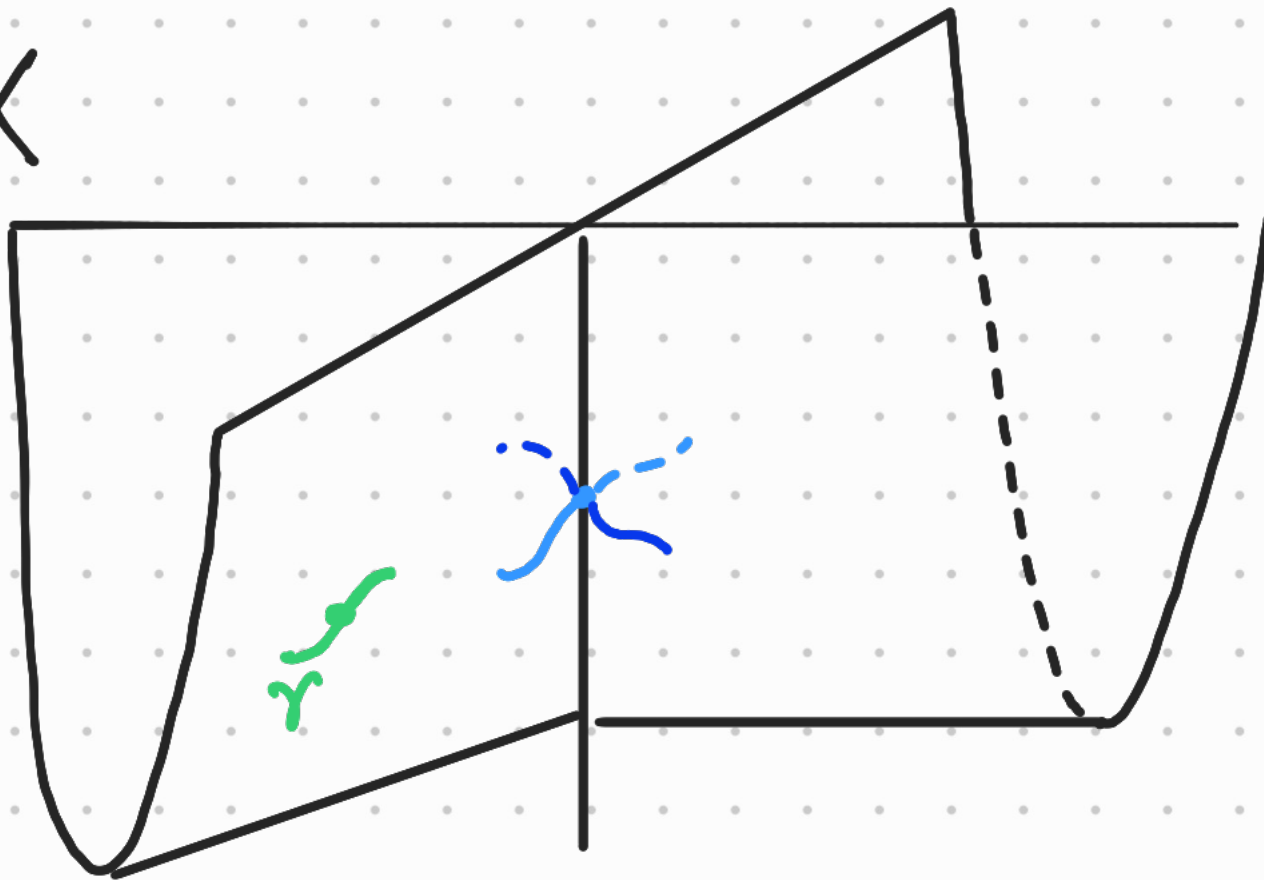
special  
dynamics

Hamiltonian  
perturbation  
~~~~~>



separate computation!

X



$L_m X$

