Drafting Workshop Talks with abstracts

Monday (29.01.2024)

9:30-10:00 **Felix Weilacher:** *Shannon's Theorem and the Unbalanced Matching Problem in the Measurable Context*

Abstract: A theorem of Shannon states that any multigraph of maximum degree d admits a proper edge coloring using at most 3d/2 colors. We investigate the status of this theorem in the setting of "descriptive" combinatorics. That is, we are interested in finding Borel, measurable, etc. edge colorings of Borel graphs on standard Borel spaces.

We focus on the measurable setting as this is where most of the work needed to be done. There, we obtain a full generalization of Shannon's result: Any Borel multigraph of maximum degree d on a standard probability space admits a Borel edge coloring using at most 3d/2 colors almost everywhere. We also prove that losing a null set is not necessary for graphs of subexponential growth rate.

The proof uses a coloring procedure first used in distributed computing by Ghaffari, Kuhn, Maus, and Uitto. In the measurable setting, the most difficult step turns out to be the following result, interesting in its own right: Let G be a Borel bipartite multigraph on a standard probability space where all vertices on one side have larger degree than all vertices on the other. Then there is a Borel matching of G covering almost all the large degree vertices. Though for measure preserving graphs this follows immediately from the well known Lyons-Nazarov theorem, the general result requires new ideas inspired by Grebik's recent proof of the measurable Vizing's theorem.

This is joint work with Anton Bernshteyn and Matt Bowen.

10:05-10:35 **Anna Roig Sanchis:** *On the length spectrum of random hyperbolic 3-manifold*

Abstract: We are interested in studying the behavior of geometric invariants of hyperbolic 3-manifolds, such as the length of their geodesics. A way to do so is by using probabilistic methods. That is, we consider a set of hyperbolic manifolds, put a probability measure on it, and ask what is the probability that a random manifold has a certain property. There are several models of construction of random manifolds. In this talk, I will explain one of the principal probabilistic models for 3 dimensions and I will present a result concerning the length spectrum -the set of lengths of all closed geodesics- of a 3-manifold constructed under this model.

11:00-11:30 **Alberto Mizrahy Campos:** *Covering distributions*

Abstract: Consider a one-dimensional discrete torus, where at each time step a random arc with some random length is placed uniformly in the space. In this talk, when changing the arc length distribution, we will give a brief introduction to the different types of limiting distributions of the covering time.

11:35-12:05 **Giulio Zucal:** *Spectral properties of the Non-backtracking Laplacian*

Abstract: Spectral theory studies the eigenvalues of operators. Studying the spectrum of particular linear operators, such as the Laplacian, is a powerful tool for exploring the geometry of both continuous and discrete spaces. We introduce the non-backtracking Laplacian of a graph, a new linear operator on a graph, and we study its spectrum. In particular, we investigate the properties of various classes of non-backtracking Laplacian

eigenfunctions and related eigenvalues. Moreover, we introduce circularly partite graphs as a generalization of bipartite graphs and we study their non-backtracking Laplacian spectrum. Using this notion, we state a sharp upper bound for the spectral gap from 1.

14:30-15:00 **Gilles Felber:** *A restriction norm problem for Siegel modular forms*

Abstract: I will first introduce Quantum Unique Ergodicity (QUE). This is a conjecture about equidistribution of Laplace eigenvalues, for examples stationary waves on a string. Then I will present my thesis project. We will consider restriction norms of Siegel modular forms to the imaginary axis and see how a result about their size relates to QUE as well as to the Generalized Lindelöf Hypothesis.

15:05-15:35 **James Taylor:** *D-modules, p-adic representations of GL_n(F), and the Drinfeld tower*

Abstract: Given a group G, we are interested in constructing its irreducible representations. These are often obtained by considering the action of G on a sufficiently rich geometric object. Our interest is in the group G = GL_n(F), where F is a finite field extension of Q_p (the p-adic numbers). By considering the action of GL_n(F) on the Drinfeld tower, a mysterious family of p-adic analytic spaces, it is possible to construct all of the irreducible supercuspidal l-adic representations of GL_n(F) (when l is not equal to p). We are interested in the significantly harder problem of understanding the p-adic representations of GL_n(F). In this talk, we discuss how we can shed light on a very natural family of such representations which also arises from the Drinfeld tower, through studying certain equivariant D-modules on the p-adic upper half plane.

16:00-16:30 **Huimin Zhang:** *Analytic twisted sums and applications*

Abstract: Bounding partial sums of arithmetic functions plays an important role in analytic number theory. In this talk, we will study the sum of GL(2)×GL(2) Fourier coefficients weighted by oscillatory exponential functions and its application in bounding partial sums of the coefficients. We will report on several cases where factorization of the coefficients allows one to improve some previous works.

16:35-17:05 **Jakob Streipel:** *Zero-density estimates via second moments*

Abstract: The Riemann Hypothesis for an L-function says that its nontrivial zeros lie on a certain line. If this were true, it simplifies lots of applications of these L-functions simply because a certain part of the calculation now becomes fixed instead of potentially variable. We don't know how to prove the Riemann Hypothesis for any of the L-functions I study, but sometimes for certain applications or computations the second best thing suffices: knowing that, in some way, at most very few of the zeros aren't on the line where they ought to be. That way, even though there might be outliers, they are rare enough to not ruin the overall picture too much. Such a result is known as a zero-density estimate.

This talk will be about zero-density estimates in general, how one might find them, and how second moments of L-functions are a natural way to do so. In doing so I will discuss recent work, joint with Sheng-Chi Liu, in which we compute a zero-density estimate for the L-functions associated with GL(2) Hecke--Maass cusp forms.

Tuesday (30.01.2024)

9:30-10:00 **Roman Gambelin:** *An extension of the algebraic Aldous diffusion*

Abstract: In this talk, we introduce a sequence of Markov chains on cladograms with a fixed number of leaves and study its limit as the number of leaves tends to infinity. We will see that, when placed in a suitable space of trees, the chains converge to a diffusion which is symmetric for the law of a stable tree. This is an extension of an open problem posed by D. Aldous in 2000 and relies on the theory of algebraic measure trees developed by A. Winter and W. Löhr in 2018 to solve this particular problem.

10:05-10:35 **Geuntaek Seo:** *A gradient flow for the Porous Medium Equations with Dirichlet boundary conditions*

Abstract: We consider the gradient flow structure of the porous medium equations with nonnegative constant boundary conditions. We prove that weak solutions to the equations can be obtained by the variational steepest descent scheme by considering an entropy functional with respect to W b2 distance, which is a modified Wasserstein distance introduced by Figalli and Gigli [J. Math. Pures Appl. 94, (2010), pp. 107-130]. In addition, we establish an energy dissipation inequality.

This is based on a joint work with Dongkwang Kim and Dowan Koo.

11:00-11:30 **Subrata Golui:** *Discrete-time zero-sum games for Markov chains with risk-sensitive average cost criterion*

Abstract: In this talk, we first investigate zero-sum stochastic games for controlled discrete time Markov chains with risk-sensitive average cost criterion with countable/compact state space and Borel action spaces. The payoff function is nonnegative and possibly unbounded for countable state space cases and for compact state space cases it is a real-valued and bounded function. For countable state space cases, under a certain Lyapunov type stability assumption on the dynamics we establish the existence of the value and a saddle point equilibrium. For compact state space cases, we establish these results without any Lyapunov type stability assumptions. Using the stochastic representation of the principal eigenfunction of the associated optimality equation, we completely characterize all possible saddle point strategies in the class of stationary Markov strategies. Then, I will discuss my research works briefly. Also, I will discuss my proposed plan.

11:35-12:05 **Ji Zeng:** *Variation of no-three-in-line problem*

Abstract: The famous no-three-in-line problem by Dudeney more than a century ago asks whether one can select 2n points from the grid [n]^2 such that no three are collinear. We present two results related to this problem. First, we give a non-trivial upper bound for the maximum size of a set in [n]^4 such that no four are coplanar. Second, we characterize the behavior of the maximum size of a subset such that no three are collinear in a random set of \mathbb{F}_q^2, that is, the plane over the finite field of order q. We discuss their proofs and related open problems.

14:30-15:00 **Jiaxi Nie:** *Sidorenko Hypergraphs and Random Tur\'an Numbers*

Abstract: Given an r-uniform hypergraph H, the random Turán number $\ex(G^r_{n,p},H)$ is the maximum number of edges in an H-free subgraph of $G^r_{n,p}$, where $G^r_{n,p}$ is the Erd\"os-R\'enyi random hypergraph. In the case when H is not r-partite, the problem has been essentially solved independently by Conlon and Gower;

and Schacht. In the case when H is r-partite, the degenerate case, not much is known.
The Sidorenko conjecture is a notorious problem in extremal combinatorics. It is known that its hypergraph analog is not true. Recently, Conlon, Lee, and Sidorenko discover a relation between Sidorenko conjecture and Turan problem.
In this talk, we introduce some recent results on degenerate random Turan problem and its relation to the hypergraph analog of Sidorenko conjecture.

15:05-15:35 **Laurentiu Ploscaru:** *Distinct degrees and homogeneous sets*

Abstract: In this talk, I will discuss some recent work examining the extremal relationship between two well-studied graph parameters: the order of the largest homogeneous set in a graph G and the maximal number of distinct degrees appearing in an induced subgraph of G, denoted respectively by hom (G) and f(G). Our main theorem improves estimates due to Bukh and Sudakov and to Narayanan and Tomon and shows that if G is an n-vertex graph with hom(G) at least n1/2 then f(G) > (n / hom (G))1 – o(1). The bound here is sharp up to the o(1)-term, and asymptotically solves a conjecture of Narayanan and Tomon. In particular, this implies that max { hom (G), f(G) } > n1/2 - o(1) for any n-vertex graph G, which is also sharp. The relationship between these parameters changes when hom (G) < n1/2. I hope to discuss the suspected relationship in this other region, along with supporting results. Joint work with Eoin Long.

16:00-16:30 **Andrea Freschi:** *Dirac-type results for tilings in vertex ordered graphs*

Abstract: A (vertex) ordered graph G is a graph with vertex set V (G) = {1, 2, . . . , |V (G)|}. Given two ordered graphs G and H, we say G contains a copy of H if there exists a graph homomorphism q : V (H) → V (G) which preserves the order of the vertices of H, i.e., q(1) < q(2) < · · · < q(|V (H)|).

A perfect H-tiling in G is a collection of disjoint copies of H in G which cover the whole vertex set of G. Balogh, Li and Treglown initiated the study of Dirac-type problems for tilings in ordered graphs. In this talk, I will present a result which asymptotically determines the minimum degree threshold for forcing a perfect H-tiling in an ordered graph (for any fixed ordered graph H). This provides an ordered graph analogue of the seminal tiling theorem of Kühn and Osthus.

This is based on joint work with Andrew Treglown.

16:35-17:05 **Simona Boyadzhiyska:** *Ramsey goodness of loose paths*

Abstract: The \emph{Ramsey number} of a pair of graphs (G,H), denoted by $R(G,H)$, is the smallest integer n such that, for every red/blue-coloring of the edges of the complete graph K_n, there exists a red copy of G or a blue copy of H. In the 1980s, Burr showed that, if G is large and connected, then $R(G,H)$ is bounded below by $(v(G)-1)(\chi(H)-1)+\sigma(H)$, where $\chi(H)$ is the chromatic number of H and $\sigma(H)$ stands for the minimum size of a color class over all proper $\chi(H)$-colorings of H. We say that G is \emph{H-good} if $R(G,H)$ is equal to this general lower bound. This notion was first studied systematically by Burr and Erd\H{o}s and has received considerable attention from researchers since its introduction. Among other results, it was shown by Burr that, for any graph H, every sufficiently long path is H-good.

These concepts generalize in the natural way to k-graphs, and in this talk we will explore the notion of Ramsey goodness when G is an ℓ-path for some $1\leq \ell \leq k-1$. We will show that, while long loose paths are not always H-good, they are

very close to being H-good for every k-graph H. As we will see, this is in stark contrast to the behavior of ℓ-paths for larger ℓ.

This is joint work with Allan Lo.

Wednesday (31.01.2024)

10:05-10:35 **Qiuting Chen:** *Bipartite walks are better than Grover's walk*

Abstract: In this talk, I will introduce a discrete quantum walk model called bipartite walks. Bipartite walk model generalizes many well-known discrete quantum walk models including Grover's walk. Grover's walk is a well-studied quantum walk model, which can be used to implement Grover's search algorithm. We show that Grover's walk can be viewed as a special case of bipartite walk. Not only that, we can also show that given a graph, a periodic bipartite walk reaches its periodic state twice as fast as Grover's walk does.

11:00-11:30 **Yuwen Wang:** *Expected hitting time estimates on finite graphs*

Abstract: The expected hitting time from vertex a to vertex b, H(a,b), is the expected value of the time it takes a random walk starting at a to reach b. In this talk, we shall discuss estimates for H(a,b) when the distance between a and b is comparable to the diameter of the graph, and the graph satisfies a Harnack condition. We show that, in such cases, H(a,b) can be estimated using a formula in terms of the volumes of balls around b. We give an outline of the proof using Green functions and heat kernel estimates. Using this result, we can then estimate H(a,b) on various graphs, such as rectangular tori, some convex traces on the integer lattice, and fractal graphs.

Joint work with Laurent Saloff-Coste.

11:35-12:05 **Clark Lyons:** *Baire Measurable Matchings in Non-amenable Graphs*

Abstract: Tutte's theorem provides a necessary and sufficient condition for a finite graph to have a perfect matching. In this talk I will present joint work with Kastner showing that if a locally finite Borel graph satisfies a strengthened form of Tutte's condition, then it has a perfect matching which is Baire measurable. As a consequence, the Schreier graph of a free action of a non-amenable group on a Polish space admits a Baire measurable perfect matching. This is analogous to the result of Csoka and Lippner on factor of IID perfect matchings for non-amenable Cayley graphs.