Abstracts of the Drafting Workshop

Jing Yu: Borel Graphs of Controlled Growth: Embeddings, Hyperfiniteness, and Local Lemma

Abstract: I will discuss recent progress on the large-scale geometry of Borel graphs with controlled growth, and explain how geometric and probabilistic methods lead to structural consequences in descriptive combinatorics.

The starting point is a theorem of Krauthgamer and Lee which shows that every connected graph of polynomial growth admits an injective contraction into $(\mathbb{Z}^n, \|\cdot\|_\infty)$ for some n. In joint work with Bernshteyn, we strengthen this in the Borel setting: we construct coarse embeddings from Borel graphs of polynomial growth into integer grids $\mathbb{Z}^d$, where d depends linearly on the asymptotic growth rate. As a consequence, graphs generated by free Borel actions of $\mathbb{Z}^d$ are universal for all Borel graphs of polynomial growth, and in particular every such Borel graph is hyperfinite. This answers a question of Marks and extends classical results of Jackson–Kechris–Louveau beyond Schreier graphs of group actions.

The proof combines padded decompositions, asymptotic dimension, and a Borel version of the Lovász Local Lemma. I will also briefly discuss ongoing work on Borel graphs of intermediate growth, where asymptotic dimension can be infinite but hyperfiniteness is still expected, and explain how similar techniques lead to partial progress.

Ce Chen: Containers and Stability in Extremal Combinatorics

Abstract: A central theme in my research is to study extremal problems where a global constraint guarantees the existence of a large substructure. In this talk I will discuss two projects along these lines.

First, for graphs F and H, let $f_{F,H}(n)$ be the minimum possible size of a largest F-free induced subgraph in an n-vertex H-free graph. This notion generalizes both Ramsey and Erdős–Rogers functions. We developed a general framework for bounding $f_{F,H}(n)$ by establishing a container lemma for F-free subgraphs in locally dense graphs. In particular, we obtained general upper bounds on $f_{F,H}(n)$ for F with small Turán number.

Second, a set system F on the ground set [n] is called a maximal k-wise intersecting family if every collection of k sets in F has non-empty intersection and F cannot be enlarged while preserving this property. Erdős and Kleitman asked for the smallest possible size of such a family. We resolved the case k=3 for large n by characterizing the unique extremal family, thus determining the exact minimum.

Shifan Zhao: Non-vanishing of Automorphic L-functions

Abstract: Let π be an automorphic representation. Let $L(s, \pi)$ be the standard L-function associated to π. The study of the non-vanishing of $L(s, \pi)$ at the center point $s = 1/2$ and near the critical point $s = 1$ are two central problems in analytic number theory with many applications in arithmetic geometry and in the theory of primes. In this talk, we will first introduce these two types of problems through a classical example: counting points on elliptic curves. We will then summarize some important known results and the ideas behind them. Finally, we will present our recent results and the key novelty in the proof.

Ramon Ivan Garcia Alvarez: Maximal independent sets in the middle two layers of the Boolean lattice

Abstract: Let Q_n denote the discrete hypercube of dimension n, defined as the graph whose vertices are the subsets of $\{1, ..., n\}$, with two vertices adjacent if and only if their corresponding sets differ in exactly one element. Let $B(2d1, d)$ be the subgraph of the hypercube Q_{2d1} induced by its two largest layers.

The aim of this talk is to study the asymptotic behavior of the number of independent sets and maximal independent sets in $B(2d - 1, d)$.

Duffus, Frankl and Rödl initiated this line of research by asking for the asymptotics of the logarithm of the number of maximal independent sets in $B(2d1, d)$. Ilinca and Kahn subsequently determined the logarithmic asymptotics and raised the question of their precise asymptotics. In this talk, we derive precise asymptotic formulas for both the number of independent sets in $B(2d1, d))$ and the number of maximal independent sets, denoted by $mis(B(2d - 1, d))$.

Attila Jung: Fractional Helly theorems

Abstract: The fractional Helly theorem of Katchalski and Liu states that if a constant fraction of (d+1)-tuples in a finite collection of d-dimensional convex sets intersect, then a constant fraction of the entire collection must share a common point. In this talk, I will present several extensions of this fundamental result. Specifically, we will explore variants concerning volumes of convex sets, monotone properties of axis-parallel boxes, and stabbing of "fat" convex sets by k-dimensional affine subspaces. I will conclude by discussing open problems concerning Helly-type theorems within the framework of abstract convexity spaces. The talk is based on joint works with Nóra Frankl, Márton Naszódi, Dömötör Pálvölgyi, and István Tomon.

Vladimir Boskovic: Dimers and Saturation

Abstract: The first part of my talk will be about the dimer model, which studies random perfect matchings on graphs. Significant progress has been made in the bipartite dimer model, particularly concerning spectral curves, integrable systems, and limit shapes. I will present recent results on the Newton polygons associated with the characteristic polynomials arising from the dimer model of certain families of non-bipartite graphs. I will also discuss the flip (or Glauber) dynamics on the non-bipartite graphs obtained by taking any 3-regular graph and replacing all of its vertices with triangles. In the second part, I will discuss saturation problems in graph theory. They are defined as analogues of the classical extremal problems ask for the minimum number of edges in a graph H on n vertices, which does not contain G as a subgraph but adding any missing edge to H creates a copy of G. Besides the usual unordered graphs, we can ask this question for graphs whose vertices or edges are ordered. I will present some partial progress toward characterizing the order of magnitude of the saturation functions in these settings. In both parts of the talk, I will emphasize the open questions and possible future directions.

Zak Smith: Factors and powers of Hamilton cycles in the budget-constrained random graph process

Abstract: Consider the following budget-constrained random graph process introduced by Frieze, Krivelevich and Michaeli. A player, called Builder, is presented with t distinct edges of K_n one by one, chosen uniformly at random. Builder may purchase at most b of these edges, and must (irrevocably) decide whether to purchase each edge as soon as it is offered. Builder's goal is to construct a graph which satisfies a certain (monotone increasing) property. In this talk, I will present the model in detail and discuss some state-of-the-art results for different properties, including new results for the containment of different F-factors or powers of Hamilton cycles. This is based on joint work with Alberto Espuny Díaz, Frederik Garbe, and Tássio Naia.

Katalin Berlow: LCLs on Grids

Abstract: We study the complexity of labelling problems on Borel graphs induced by actions of Z^n. Our results separate various complexity classes that were not previously known to be distinct. In particular, we find a problem which has a measurable solution but no Baire measurable or Borel solution.

Yuxin Zhang: Quantum singular value transformation without block encodings

Abstract: Quantum singular value transformation (QSVT) is a unifying framework that encapsulates most known quantum algorithms and serves as the foundation for new ones. However, existing implementations typically rely on block encoding, incurring an intrinsic O(log L) ancilla overhead for polynomial transformations of a Hamiltonian expressed as a linear combination of L terms. In this talk, I will introduce new quantum algorithms for implementing QSVT without block encodings, based on our recent work [arXiv:2504.02385, arXiv:2510.06851]. We propose a simple yet powerful approach that utilizes basic Hamiltonian simulation techniques, namely Trotterization, to: (i) eliminate the need for block encoding, (ii) reduce the ancilla overhead to only a single qubit, and (iii) still maintain near-optimal circuit complexity. Furthermore, we propose optimal randomized QSVT algorithms for cases where only sampling access to the Hamiltonian terms is available. As applications, we develop end-to-end quantum algorithms for quantum linear systems and ground state property estimation, both achieving near-optimal complexity without relying on oracular access.