ABSTRACTS

Drafting Workshop in Discrete Mathematics and Probability 2025

Laurel Rainie Heck Bozzai

Title: *Vector Sum Results*

In this talk I will present a diverse collection of my results relating to two vector sum problems from convex and discrete geometry: the vector balancing problem and the Steinitz problem. I will focus on two particular results of interest: a colorful generalization of classic vector balancing results and a reduction of the Euclidean Steinitz problem to almost-unit vectors for the case of the Euclidean norm. In addition, I will briefly discuss applications of such problems to concrete tasks in machine learning.

Kenneth Moore

Title: *Arithmetic Progressions in Euclidean Ramsey Theory*

In this talk we discuss recent constructions in Euclidean Ramsey theory. A question is: for what values of s and t is there a red/blue colouring of N-dimensional Euclidean space with no red s-term and no blue t-term unit-separated arithmetic progressions? In particular, if s is small (say, < 6), how small may we take t? The case of s=3 was previously studied by Conlon and Wu, who found a colouring with t=2^10. Recently, Führer and Tóth improved this to t=1177, and we build upon their work to give a construction with t=20. This talk is based on a joint work with Gabriel Currier and Chi Hoi Yip.

Mehmet Akif Yildiz

Title: *Path decompositions of oriented graphs*

We consider the problem of decomposing the edges of a directed graph into as few paths as possible. There is a natural lower bound for the number of paths in any path decomposition dictated by vertex degree imbalances, and any directed graph that meets this bound is called _consistent_. In 1976, Alspach, Mason, and Pullman conjectured that (as a generalization of Kelly's conjecture on Hamilton decompositions of regular tournaments) every tournament of even order is consistent. This conjecture was recently verified for large tournaments by Girão, Granet, Kühn, Lo, and Osthus. A stronger conjecture, proposed by Pullman in 1980, states that every orientation of every regular graph with odd degree is consistent. In this talk, I will present our recent work (joint with Viresh Patel) that establishes Pullman's conjecture for the cases of _random_ regular graphs and regular graphs _without short cycles_.

Krishnendu Bhowmick

Title: *Distance Problems in the Plane*

The Distinct Distance Problem and the Unit Distance Problem are among the "_most striking contributions_" of Erdős to discrete geometry, introduced in 1946. Since then, numerous closely related problems have been formulated, extending to questions about distances in higher dimensions, other fields, or even in other norms.... Similar questions about other geometric structures (insead of distances) are also considered for example triangles or angles.

Many tantalizing related questions arise, some of which connect to other branches of mathematics, such as Additive Combinatorics. However, due to time constraints, this talk will focus on a selection of elementary questions about the repetition of distances in the Euclidean plane. For instance, what happens when we restrict the set of points to be in a convex position? Or, what if the point set excludes collinear triples? It may come as a surprise, but most of the best-known bounds for these problems are based on simple, and elegant arguments, reminiscent of the ones found in "_The Book_."

Erfei Yue

Title: *Results on Bollob´as set-pair systems*

Suppose $\mathcal{P} = \{(A_i, B_i) \mid i \in [m]\}$ is a family of pairs of sets, where $A_i, B_i \subseteq [n]$, and $A_i \cap B_i = \emptyset$. Then $\mathcal{P}$ is called a Bollobás system if $A_i \cap B_j \neq \emptyset$ when $i \neq j$, and a skew Bollobás system if $A_i \cap B_j \neq \emptyset$ when $i < j$.

In 1965, to solve a problem on hypergraphs, Bollobás [2] proved that for a Bollobás system $\mathcal{P} = \{(A_i, B_i) \mid i \in [m]\}$, we have $\sum_{i=1}^{m} \binom{|A_i|+|B_i|}{|A_i|}^{-1} \leqslant 1$. If we further requests $|A_i| = a, |B_i| = b$ for every i, this inequality shows that the maximum size of the Bollobás system is $\binom{a+b}{a}$. In 1982, Frankl [3] proved that $\binom{a+b}{a}$ is also the maximum size of a skew Bollobás system.

However, for the nonuniform case, suppose $\mathcal{P} = \{(A_i, B_i) \mid i \in [m]\}$ is a skew Bollobás system, 1 is no longer the maximum value of $\sum_{i=1}^{m} \binom{|A_i|+|B_i|}{|A_i|}^{-1}$. In 2023, Hegedüs and Frankl [4] proved that this maximum value is $1 + n$, provided $A_i, B_i \subseteq [n], \forall i \in [m]$. Using the argument of random permutation, we improve their result to

$$\sum_{i=1}^{m} \left((1 + |A_i| + |B_i|) \binom{|A_i| + |B_i|}{|A_i|} \right)^{-1} \leqslant 1.$$

In 1985, Alon [1] generalized the uniform version of Bollobás Theorem to partitions, and determined the maximum size of the family. As a variation, we considered the nonuniform case, and proved two inequalities for Bollobás systems and skew Bollobás systems.

A natural generalization of (skew) Bollobás systems involves considering families of d-tuples instead of set-pairs. Hegedüs and Frankl [4] recently extended the concept of Bollobás systems to d-tuples, conjecturing that for a Bollobás system of d-tuples, $\{(A_i^{(1)}, \ldots, A_i^{(d)}) \mid i \in [m]\}$, the maximum value of $\sum_{i=1}^{m} \binom{|A_i^{(1)}|+\cdots+|A_i^{(d)}|}{|A_i^{(1)}|,\ldots,|A_i^{(d)}|}^{-1}$ is also 1. We refutes this conjecture and establishes an upper bound for the sum. In the case $d = 3$, the derived upper bound is asymptotically tight. Furthermore, we sharpen an inequality for skew Bollobás systems of d-tuples in [4], We also determine the maximum size of a uniform (skew) Bollobás system of d-tuples on both sets and spaces.

Alex Malekshahian

Title: *On the typical structure of antichains in the Boolean lattice*

An old question of Dedekind asks for the number of antichains (monotone Boolean functions) in the Boolean lattice on n elements. After a long series of increasingly precise results, Korshunov determined this number up to a multiplicative factor of (1+o(1)). We revisit Dedekind’s problem and study the typical structure of antichains using tools from probability and statistical physics. This yields a number of results which include refinements of Korshunov’s asymptotics, asymptotics for the number of antichains of a fixed size, and a 'sparse' version of Sperner’s theorem. Joint work with Matthew Jenssen and Jinyoung Park.

Raz Firanko

Title: *Quantum Markov Processes and their Steady States.*

We study quantum generalizations of Markov processes with local transition rules and their density-matrix fixed points, also known as steady states. Specifically, we focus on quantum maps acting on many-particle systems arranged on a graph, which admit a Kraus decomposition in terms of local operators. These maps serve as quantum analogs of classical Markov processes and can be implemented on a quantum computer.

We address the question of when these fixed points correspond to Gibbs measures (equilibrium states) of a local Hamiltonian. By perturbatively interpolating from a 1-local map to a 2-local map, we show that the fixed-point Gibbs Hamiltonian perturbs into a quasi-local Hamiltonian, where the range of the Hamiltonian terms grows with the perturbative order.

Our result is established using a multi-parameter perturbation framework that respects the geometric structure of the system. In this talk, I will provide an introduction to the key concepts and questions, present partial results, and outline the proof of the main theorem.