Abstract Booklet

Winter Workshop:
Fourier Analysis and Its Applications

Erdős Center

January 23, 2024

Contents

Invited Talks

Metric Fourier approximation of set-valued functions of bounded variation

29 Jan
10.45

Elena Berdysheva

University of Cape Town

We study Fourier approximation of set-valued functions (SVFs, multifunctions) mapping a compact interval $[a, b]$ into the space of compact nonempty subsets of $\mathbb{R}^d$.

Older works on approximation of SVFs consider almost exclusively SVFs with convex values. The standard techniques used for work with SVFs were developed for convex sets and suffer from the phenomenon called convexification. As a result, corresponding approximation methods deliver approximants whose values are convex, even if the function to be approximated did not have this property. Clearly, such methods are useless when one wants to approximate a set-valued function with general, not necessarily convex values.

A pioneering work on approximation of SVFs with general values was done by Z. Artstein who constructed piecewise linear approximantis based of special pairs of points that are termed in later works "metric pairs". Using the concept of metric pairs, N. Dyn, E. Farkhi and A. Mokhov developed in a series of works techniques that are appropriate for work with SVFs with general, not necessarily convex values.

In this talk I will describe a construction that adapts the trigonometric Fourier series to set-valued functions with general (not necessarily convex) compact images. We recover the classical Dirichlet-Jordan Theorem for functions of bounded variation.

We are not aware of other attempts to develop Fourier series for SVFs.

Joint work with Nira Dyn, Elza Farkhi and Alona Mokhov (Tel Aviv, Israel).

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2 Febr 11:30

Convergence and growth of dilated series $\sum c_k f(kx)$

Istvan Berkes
Rényi Institute of Mathematics

The basic object of harmonic analysis is the trigonometric system, but the question of convergence and asymptotic properties of sums $\sum c_k f(kx)$ for general periodic functions f has been raised as early as in 1910. The behavior of such sums can be very different from the trigonometric case: for example, Wintner showed that for periodic $f \in L^2$ with mean 0 the sum $\sum_{k=1}^{\infty} c_k f(kx)$ converges in L^2 norm for all (c_k) with $\sum c_k^2 < \infty$ iff the Dirichlet series $\sum_{k=1}^{\infty} a_k k^{-s}$ and $\sum_{k=1}^{\infty} b_k k^{-s}$, with a_k and b_k denoting the Fourier coefficients of f, converge and represent a bounded function in the half plane $\Re(s) > 0$. Also, for smooth periodic f the central limit theorem holds for the sequence $f(2^k x)$, but this breaks down for $f((2^k - 1)x)$. This shows that the asymptotic theory of sums $\sum c_k f(n_k x)$ is intimately connected with number theory, leading to a highly interesting theory exhibiting many deep results and open problems. The purpose of this talk is to give a survey of this theory and to discuss recent developments.

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31 Jan 10:45

Hankel operators, div-curl lemma and weak factorization: endpoint estimates

Aline Bonami
Orléans University

We start from the the fact that the pointwise product of a function in $H^1(\mathbb{R}^n)$ and a function in $BMO(\mathbb{R}^n)$ can be given a meaning in the distribution sense and belongs to the space $L^1(\mathbb{R}^n) + H_{\log}(\mathbb{R}^n)$. Here $H_{\log}(\mathbb{R}^n)$ is the space of tempered distributions f such that their maximal function $\mathcal{M}_\varphi(f)$ belongs to the space

$$L_{\log}(\mathbb{R}^n) := \{\text{measurable } g; \int_{\mathbb{R}^n} \frac{|g|}{1 + \log_+(|g|) + \log_+(|x|)} dx < \infty\}.$$

We interest ourselves to the converse problem: when can a function in $H_{\log}(\mathbb{R}^n)$ be written in terms of products of a function in $H^1(\mathbb{R}^n)$ and a function in $BMO(\mathbb{R}^n)$? We show how an answer is given in dimension 1 by considering an analogous problem for holomorphic functions. In higher dimension we use the converse of an adapted div-curl lemma to obtain a weak factorization.

Finally, coming back to the complex setting, we show how the factorization can be used to obtain estimates with loss for the Hankel operator.

Generalized Fourier quasicrystals, zeros of Dirichlet series, and almost periodic sets

2 Febr 9:30

Sergii Favorov

Kharkiv University

Definitions of a crystal measure and a Fourier quasicrystal are given, and conditions are shown when the crystal measure is periodic. We then discuss the relationship between crystal measures with unit masses and almost periodic sets, as well as the connection between such crystal measures and the zero sets of exponential polynomials or absolutely convergent Dirichlet series with bounded spectrum.

Quantum Harmonic Analysis via the Banach Gelfand Triple

31 Jan 9:30

Hans G. Feichtinger

University of Vienna

It is the purpose of this talk to demonstrate that the *Banach Gelfand Triple* (**BGT** or *rigged Hilbert space*) (S_0, L^2, S_0^*), consisting of the Segal Algebra $S_0(R^d)$, the Hilbert spaces $L^2(R^d)$ and the dual space, the space of so-called *mild distributions* allows to describe the key ideas of the fundamental paper of R. Werner ([1]) in a relatively simple way. The Fourier transform preserves this BGT.

In fact, the BGT setting allows to describe linear operators T by their kernels $\kappa(T) \in S_0^*(R^{2d})$, but in a similar way by their Kohn-Nirenberg symbol (KNS-symbol) $\sigma(T)$ or their *spreading function* $\eta(T)$. For Hilbert-Schmidt operators these relations are in fact unitary isomorphism to $L^2(R^{2d})$, and $\eta(T)$ is the symplectic Fourier transform of $\kappa(T)$, see [2] and [3].

Using these identification of operators with corresponding functions or mild distributions on phase space it is possible to redefine (in an equivalent way) the two concepts of convolution as introduced by R. Werner essentially as "ordinary convolution" at the KNS-level, combined with the identifications mentioned so far. Various identities, but also associativity or the operator Fourier transform are easily derived in this setting.

Based on the approximation (in the sense of the S_0-norm) of either kernels or symbols of these operators it is then possible to prepare the ground for numerical (FFT-based) approximate computations, making use of the corresponding, discrete versions of all the expressions. This makes sense because the theory can be developed in the context of general locally compact Abelian groups, thus including the finite Abelian groups as special case.

Overall this is new material which is currently worked out in detail together with several young colleagues.

[1] R. F. Werner. Quantum harmonic analysis on phase space. *J. Math. Phys.*, 25(5):1404–1411, 1984.

[2] H. G. Feichtinger and W. Kozek. Quantization of TF lattice-invariant operators on elementary LCA groups. In H. G. Feichtinger and T. Strohmer, editors, *Gabor analysis and algorithms*, Appl. Numer. Harmon. Anal., pages 233–266. Birkhäuser, Boston, MA, 1998.

[3] E. Cordero, H. G. Feichtinger, and F. Luef. Banach Gelfand triples for Gabor analysis. In *Pseudo-differential Operators*, volume 1949 of *Lect. Notes Math.*, pages 1–33. Springer, Berlin, 2008.

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30 Jan
10:45

Almost everywhere convergence of means of subsequences of partial sums of trigonometric (and other) Fourier-series

György Gát

University of Debrecen

In this talk, we give a résumé concerning some recent results in the theory of summation of trigonometric Fourier-series. We discuss the case when only a subsequence of the sequence of partial sums is given ([2,3]). We examine for which index sequences and in what sense the original function can be reconstructed. For convex index sequences and continuous functions (supremum norm) Carleson [1] has given a necessary and sufficient condition. His result is partly based on two theorems of Kahane and Katznelson [4]. For integrable functions and almost everywhere convergence, the situation is considerably more difficult. We also formulate some problems and conjectures in this research area of Fourier analysis.

[1] L. Carleson, *Appendix to the paper by J.P. Kahane and Y. Katznelson*, Series de Fourier des fonctions bornees, Studies in pure mathematics, Birkhauser, Basel-Boston, Mass. (1983), 411–413.

[2] G. Gát, *Cesàro means of subsequences of partial sums of trigonometric Fourier series*, Constructive Approximation, **49** (2019), no. 1, 59–101.

[3] G. Gát, *Almost everywhere divergence of Cesàro means of subsequences of partial sums of trigonometric Fourier series*, Mathematische Annalen (doi.org/10.1007/s00208-023-02746-z).

[4] J.P. Kahane et Y. Katznelson, *Series de Fourier des fonetions bornées*, Series de Fourier des fonctions bornees, Studies in pure mathematics, Birkhauser, Basel-Boston, Mass. (1983), 395–410.

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t-design curves and mobile sampling on the sphere

30 Jan
14:30

Karlheinz Gröchenig
University of Vienna

A spherical t-design curve is a curve on the d-dimensional sphere such that the corresponding line integral integrates polynomials of degree t exactly. Spherical t-design curves can be used for mobile sampling and reconstruction of functions on the sphere.

(i) We derive lower asymptotic bounds for the length of t-design curves.

(ii) For the unit sphere in $\mathbb{R}^3$ and small degrees, we present examples of t-design curves with small t.

(iii) We prove the existence of asymptotically optimal t-design curves in the Euclidean 2-sphere. This construction is based on and uses the existence of t-design points verified by Bondarenko, Radchenko, and Viazovska (2013). For higher-dimensional spheres we inductively prove the existence of t-design curves.

More generally, one can study the concept of t-design curves on a compact Riemannian manifold. This means that the line integral along a t-design curve integrates "polynomials" of degree t exactly. For the d-dimensional tori, we construct t-design curves with asymptotically optimal length.

This is joint work with Martin Ehler and Clemens Karner.

[1] A. Bondarenko, D. Radchenko, and M. Viazovska. Optimal asymptotic bounds for spherical designs. *Ann. Math.*, 178(2):443–452, 2013.

[2] M. Ehler, K. Gröchenig. Forum of Mathematics, Sigma. 2023;11:e105. doi:10.1017/fms.2023.106

Restriction theory, uncertainty principles and signal recovery

1 Febr
9:30

Alex Iosevich
University of Rochester

We are going to discuss a variety of uncertainty principles, some old and some new, and applications of these principles to the problem of signal recovery. Connections with some classical problems in harmonic analysis will come up as well. Most of the talk is joint work with Azita Mayeli.

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1 Febr
11:30

Connectifying counterexamples

Mihalis Kolountzakis
University of Crete

Recently Greenfeld and Tao found an example of a finite subset in $\mathbb{Z}^d$ (for some large d) which tiles $\mathbb{Z}^d$ by translations but only aperiodically, thus disproving the so-called *Periodic Tiling Conjecture* in high enough dimension.

Roughly 20 years ago the Fuglede (or Spectral set) conjecture was disproved by Tao (in the spectral implies tiling direction) and by Kolountzakis and Matolcsi (in the tiling implies spectral direction). In this problem the dimension d eventually got down to 3 for both directions.

In both these problems (aperiodicity and Fuglede conjecture) the examples found are highly dispersed subsets of $\mathbb{Z}^d$. In this work we show how to modify these examples to obtain (pathwise) *connected* subsets of $\mathbb{Z}^d$ as examples by increasing the dimension d.

This is joint work with Rachel Greenfeld.

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2 Febr
10:45

Observation of the linear Zakharov–Kuznetsov equation

Vilmos Komornik
University of Strasbourg

We report on a joint work with A. Pazoto and R. Capistrano-Filho. The Zakharov–Kuznetsov equation is a widely studied 2D generalization of the KdV equation. We study the linearized equation with periodic boundary conditions. Employing some tools from nonharmonic Fourier series we obtain several internal observability theorems.

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29 Jan
11:30

The discrete Pompeiu problem on the plane: some new results

Miklós Laczkovich
Eötvös Loránd University

We say that a finite subset E of the Euclidean plane $\mathbb{R}^2$ has the discrete Pompeiu property (dPp) if, whenever $f\colon \mathbb{R}^2 \to \mathbb{C}$ is such that the sum of the values of f on any congruent copy of E is zero, then f is identically zero. It is easy to see that every set having at most three elements has dPp. On the other hand, no finite set is known which does not have dPp. Applying harmonic analysis in some varieties connected to the problem and also some results of Euclidean Ramsey theory, it was proved by Cs. Vincze and the present authors in 2018 that every parallelogram and every quadrangle with rational coordinates has dPp. Improving upon the previous methods and using also results on linear equations of units we prove that every set having at most six elements has dPp, and every finite set consisting of algebraic numbers has dPp. We also discuss the connections with some problems concerning colorings of the plane.

This is a joint work with G. Kiss.

Schauder frames of translates in $L^p(\mathbb{R})$

30 Jan
11:30

Nir Lev
Bar-Ilan University

Does there exist a basis in the space $L^p(\mathbb{R})$ consisting of translates of a single function? I will survey the known results around this problem and present some recent work joint with Anton Tselishchev.

Quantitative bounds for the eigenvalue distribution of the spatio-spectral limiting operators

1 Febr
10:45

Azita Mayeli
City University of New York

The spatio-spectral limiting operator is the natural analog of the time-frequency limiting operator in one dimension extended to higher dimensions. In one dimension, the eigenfunctions of the time-frequency limiting operator are the prolate spheroidal wave functions. These functions are bandlimited, maximizing the L^2 norm on a specified time interval, and are used in a variety of analytical and numerical applications.

In this talk, we present an extension of an important aspect of one-dimensional analysis to spatio-spectral limiting operators in arbitrary dimensions, where one of the domains – either spatial or spectral – is a hypercube, and the other domain satisfies a symmetry condition. We estimate the distribution of eigenvalues for such operators. This is a joint work with Arie Israel.

Lower bounds of L^1-norms of non harmonic trigonometric polynomials.

1 Febr
14:30

Jaming Philippe
University of Bordeaux

In this talk we will present quantitative lower bounds of the L^1 norm of a non-harmonic trigonometric polynomial of the following form:

– let $T > 1$;
– let $(\lambda_j)_{j\geq 0}$ be a sequence of non-negative real numbers with $\lambda_{j+1} - \lambda_j \geq 1$
– let $(a_j)_{j=0,\ldots,N}$ be a finite sequence of complex numbers
then

$$\frac{1}{T}\int_{-T/2}^{T/2}\left|\sum_{j=0}^{N} a_j e^{2i\pi\lambda_j t}\right| \mathrm{d}t \geq C(T)\sum_{j=0}^{N}\frac{|a_j|}{j+1}$$

where $C(T)$ is an explicit constant that depends on T only.

The L^2 analogue is Inham's Inequality and the harmonic case (λ_j integers) is McGehee, Pigno, Smith's solution of the Littlewood conjecture while the version with non explicit constant is due to Nazarov.

This is joint work with K. Kellay and C. Saba (U. Bordeaux)

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29 Jan
14:30

Characterization of locally compact Abelian groups having spectral synthesis

László Székelyhidi
University of Debrecen

We introduce the concept of localizability of an ideal in the Fourier algebra of a locally compact Abelian group which roughly means that if the Fourier transform of a compactly supported measure is annihilated by all derivations which annihilate an ideal at its roots, then the measure itself belongs to that ideal. We show that synthesizability of a variety is equivalent to the localizability of the ideal formed by the Fourier transforms of the annihilating ideal of the variety. We utilize this result to prove that spectral synthesis holds on a compactly generated locally compact Abelian group if and only if it is topologically isomorphic to $\mathbb{R}^a \times \mathbb{Z}^b \times C$, where a, b are nonnegative integers with $a \leq 1$ and C is a compact Abelian group. Further, spectral synthesis holds on a locally compact Abelian group G with the subgroup of compact elements B if and only if G/B is topologically isomorphic to $\mathbb{R}^a \times D$, where $a \leq 1$ is a nonnegative integer, and D is a discrete torsion free Abelian group of finite rank.

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29 Jan
9:30

Polynomial inequalities and discretization problems

Sergey Tikhonov
Autonomous University of Barcelona

We survey recent developments in a study of the following classical inequalities for polynomials: Bernstein, Nikolskii, Remez as well as their connections to Marcinkiewicz–Zygmund discretization.

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30 Jan
9:30

Summability of higher dimensional Fourier series and Lebesgue points

Ferenc Weisz
Eötvös Loránd University

We consider summability methods for higher dimensional Fourier series. We focus on the Cesàro and Riesz methods and the ℓ_q- and rectangular summability. We study the norm and almost everywhere convergence. We characterize the set of the almost everywhere convergence as different types of Lebesgue points.

Oscillatory integrals and exponential sums: A unified theory

31 Jan
11:30

James Wright
University of Edinburgh

Here we consider oscillatory integrals defined over general locally compact fields $\mathbb{K}$. When $\mathbb{K} = \mathbb{R}$ is the real field, oscillatory integrals are a basic object of study in Fourier analysis. On the other hand, complete exponential sums can be realised as oscillatory integrals over the p-adic field $\mathbb{K} = \mathbb{Q}_p$. In both cases, for real oscillatory integrals and complete exponential sums, there is an extensive literature giving sharp bounds for these oscillating entities.

In this talk, we discuss a unified theory for oscillatory integrals defined over any locally compact field. This is joint work with Gian Maria Dall'Ara

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On commutative multiplicative unitaries

2 Febr
14:30

László Zsidó
Università di Roma "Tor Vergata"

Let G be a locally compact group, and m a right Haar measure on it. Then a unitary operator $V_G : L^2(G, m)\overline{\otimes}L^2(G, m) \longrightarrow L^2(G, m)\overline{\otimes}L^2(G, m)$ can be defined, which verifies two identities: the "pentagon equation", encoding the associativity of the group operation of G, and another identity, originating from the commutativity of the pointwise multiplication of scalar functions on G.

For H a complex Hilbert space, a unitary operator $V : H\overline{\otimes}H \to H\overline{\otimes}H$, satisfying the "pentagon equation", is called *multiplicative unitary.* If V satisfies also the second identity, then one says that it is a *commutative multiplicative unitary.*

It turns out that any commutative multiplicative unitary is, up to unitary equivalence and multiplicity, equal to V_G for a unique locally compact group G. In the case of separable H, this was shown in 1993 by S. Baaj and G. Skandalis. In their proof the separability is essential because direct integral theory is used. We sketch here a proof in the general case, which is more operator theoretical and is simpler even in the separable case.

The talk is based on joint work with Mauro Ricci.

Contributed Talks

On matrix transform means of Walsh-Fourier series

30 Jan
16:25

István Blahota

University of Nyíregyháza

Matrix transform (or T) means are common generalizations of Fourier partial sums and some well-known Fourier means, namely Fejér, Cesàro, Nörlund, weighted and several other ones.

The topic of this lecture are some new results with respect to matrix transform means of Walsh-Paley series of functions in L_p $(1 \leq p \leq \infty)$.

A random sampling approach for shift-invariant spaces via non-uniform periodic patterns

30 Jan
15:15

Diana Carbajal

University of Vienna

This talk presents a random sampling strategy for multi-variate signals spanned by the integer shifts of generating functions with distinct frequency profiles. We show that taking the samples over a random non-uniform periodic set produces a sampling set with high probability provided that the density of the sampling pattern exceeds the number of frequency profiles by a logarithmic factor. The result includes, in particular, the case of Paley-Wiener spaces with multi-band spectra. While in this well-studied setting delicate constructions provide sampling strategies that meet the information-theoretic benchmark of Shannon and Landau, the sampling pattern that we consider provides, at the price of a logarithmic oversampling factor, a simple alternative that is accompanied by favorable a priori stability margins (snug frames).

This is a joint work with Jorge Antezana (Autonomous University of Madrid) and José Luis Romero (University of Vienna).

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29 Jan
16:25

Polynomial equations for additive functions

Eszter Gselmann
University of Debrecen

The so-called polynomial equations play an important role both in algebra and in the theory of functional equations. In some specific cases, according to classical results, the unknown additive functions are homomorphisms, (higher-order) derivations, or linear combinations of these.

By polynomial equation of additive functions, we mean

$$P(f_1^{r_1}(x^{s_1}), \ldots, f_n^{r_n}(x^{s_n})) = 0,$$

where $P\colon \mathbb{C}^n \to \mathbb{C}$ is a n-variable polynomial, r_i, s_i are positive integers and f_i are unknown additive functions. As a matter of fact, the general solution of the above equation is not suitable to describe solutions as special functions. We illustrate this fact by considering the following three equations that are studied in the following sequence of studies.

$$\begin{aligned} \sum_{i=1}^{n} f_i(x^{p_i})g_i(x^{q_i}) &= 0, \\ \sum_{i=1}^{n} f_i(x^{p_i})g_i(x)^{q_i} &= 0, \qquad (x \in \mathbb{F}) \\ \sum_{i=1}^{n} f_i(x)^{p_i}g_i(x)^{q_i} &= 0, \end{aligned}$$

In this talk, the most impressive equation, namely

$$\sum_{i=1}^{n} f_i(x^{p_i})g_i(x^{q_i}) = 0 \qquad (x \in \mathbb{F})\,, \tag{1}$$

will be studied from this list with a fruitful theoretical description. Under some natural conditions equation (1) can be satisfied by the composition of (higher-order) derivations and homomorphisms. The purpose of the talk is to the converse by finding proper characterization of the solutions of (1) in the class of additive functions.

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29 Jan
16:00

Pego-type theorems in different convolution structures

Ágota P. Horváth
Budapest University of Technology and Economics

The Kolmogorov-Riesz characterization of compact sets in certain L^p_μ spaces contains two conditions, namely the function set has to be equivanishing and equicontinous. Pego made the remarkable observation that the two conditions can be transformed into each other by Fourier transformation. We discuss similar results in different convolution structures.

Reconstructing geometric objects from the measures of their intersections with test sets

2 Febr 15:15

Tamás Keleti

Eötvös Loránd University

Let us say that an element of a given family $\mathcal{A}$ of subsets of $\mathbb{R}^d$ can be reconstructed using n test sets if there exist $T_1, \ldots, T_n \subset \mathbb{R}^d$ such that whenever $A, B \in \mathcal{A}$ and the Lebesgue measures of $A \cap T_i$ and $B \cap T_i$ agree for each $i = 1, \ldots, n$ then $A = B$. Our goal will be to find the least such n.

We prove that if $\mathcal{A}$ consists of the translates of a fixed reasonably nice subset of $\mathbb{R}^d$ then this minimum is $n = d$. In order to obtain this result, on the one hand we reconstruct a translate of a fixed absolutely continuous function of one variable using 1 test set. On the other hand, we prove that under rather mild conditions the Radon transform of the characteristic function of K (that is, the measure function of the sections of K), $(R_\theta \chi_K)(r) = \lambda^{d-1}(K \cap \{x \in \mathbb{R}^d : \langle x, \theta \rangle = r\})$ is absolutely continuous for almost every direction θ. These proofs are based on techniques of harmonic analysis.

Joint work with Márton Elekes and András Máthé.

On the spectral synthesis for the unit circle in $\mathcal{F}L^q_s(\mathbf{R}^2)$

29 Jan 15:15

Masaharu Kobayashi

Hokkaido University

Let $\mathcal{F}L^q_s(\mathbf{R}^2)$ denote the set of all tempered distributions $f \in \mathcal{S}'(\mathbf{R}^2)$ such that the norm $\|f\|_{\mathcal{F}L^q_s} = (\int_{\mathbf{R}^2} (|\mathcal{F}[f](\xi)|(1 + |\xi|)^s)^q d\xi)^{\frac{1}{q}}$ is finite, where $\mathcal{F}[f]$ denotes the Fourier transform of f. We investigate the spectral synthesis for the unit circle $S^1 \subset \mathbf{R}^2$ in $\mathcal{F}L^q_s(\mathbf{R}^2)$. This is joint work with Prof. Sato (Yamagata University).

The uniqueness problem in phase retrieval

1 Febr 16:00

Lucas Liehr

University of Vienna

We investigate the determination of a square-integrable function from samples of the absolute value of its short-time Fourier transform (STFT), commonly known as the uniqueness problem in STFT phase retrieval. We demonstrate that if the samples are given on a lattice, then uniqueness fails, regardless of the lattice density. This stands in contrast to ordinary sampling results. Building on this non-uniqueness statement, we propose a potential solution: sampling on specific perturbations of a lattice ensures uniqueness of the phase retrieval problem. The latter yields the first uniqueness result through sampling on point sets that exhibit finite density.

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1 Febr
16:50

Weak tiling by polytopes in R^d

Matolcsi Máté

Rényi Institute of Mathematics

We say that a set A of positive Lebesgue measure weakly tiles its complement A^c, if there exists a positive, locally finite Borel measure μ on R^d such that $1_A * \mu = 1_{A^c}$. We prove that if a (not necessarily convex) polytope A weakly tiles its complement by translations, then A is equidecomposable by translations to a cube of the same volume.

Together with earlier results this enables us to conclude that if A is a convex body in R^d such that A can weakly tile its complement by translations, then A must be a convex polytope which can also tile the space properly by translations.

Joint work with M. Kolountzakis and N. Lev.

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30 Jan
16:50

Convergence in norm of Walsh matrix transform means

Károly Nagy

Eszterházy Károly Catholic University

In the presented talk we discuss the matrix summability of the Walsh-Fourier series. In particular, we discuss the convergence of matrix transform means in L_1 space and in C_W space in terms of modulus of continuity and matrix transform variation [1,2,3]. The results are related to the corresponding Lebesgue constant of matrix transformations. In the paper [4], we proved two-sides estimates for Lebesgue constants of matrix transform mean. Moreover, we show the sharpness of our result.

[1] Blahota, I. and Nagy, K., *Approximation by Θ-means of Walsh-Fourier series*, Analysis Mathematica, 44 (1), (2018) 57-71.

[2] Blahota, I. and Nagy, K., *Approximation by matrix transform of Vilenkin-Fourier series*, Publicationes Mathematicae Debrecen 99(1-2), (2021) 223-242.

[3] Blahota, I., Nagy, K. and Salim, M., *Approximation by Θ-means of Walsh-Fourier series in dyadic Hardy spaces and dyadic homogeneous Banach spaces*, Analysis Mathematica 47(2), (2021), 285-309.

[4] Goginava, U. and Nagy, K., *Matrix summability of Walsh–Fourier series*, Mathematics, 10(14), Paper: 2458, (2022) 1-25.

The relation of wavelets to Bolyai geometry

29 Jan
16:50

Margit Pap
University of Pécs

János Bolyai was one of the most original mathematicians of all time. On the proposal of the Hungarian National Commission for UNESCO, UNESCO has declared 2023 the Bolyai Memorial Year. Two hundred years ago (on 3 of November 1823) he wrote to his father Farkas Bolyai : "I created a new, different world out of nothing."

He boldly rejected the Euclidean axiom of parallelism; on the basis of a new axiom of parallelism, he first sketched the hyperbolic geometry and then developed what he called absolute geometry. His seminal work on absolute geometry, the Appendix, was published in Latin in 1832 as an appendix to the first volume of Farkas Bolyai's textbook Tentamen. Later the Appendix has been translated into several European languages.

Nikolai Ivanovich Lobachevsky had also developed almost simultaneously and independently the geometry based on the negation of the axiom of parallelism. Lobachevsky was recognized by Gauss in his life, but Bolyai was recognised only posterior.

János Bolyai's and Lobachevsky's hyperbolic geometry had also laid the mathematical foundations for Einstein's theory of gravity, the general theory of relativity. It turned out that hyperbolic geometry has many applications in cosmology, computer science and network theory, complex networks, geometric group theory, visualization of large datasets and complex structures, quantum information theory, in the study of neural networks and brain connectivity, etc.

The Poincaré disc model of hyperbolic geometry is a 2-dimensional hyperbolic geometry in which all points are inside the unit disk, and straight lines are either circular arcs contained within the disk that are orthogonal to the unit circle or diameters of the unit circle. The congruence transformations of the Poincaré disc model can be described using the Blaschke functions . These functions play a prominent role not only in complex function theory but also in control theory.

In this talk I present the relationship between wavelets and the Poincaré disc model of hyperbolic geometry. Starting from congruence transformations of the Poincaré disc model we introduced hyperbolic wavelet transformations (HWT). These transformations can be described by representations of the Blaschke group. We have started this research in 2005 joint with Ferenc Schipp and since then severel results were publishrd in this topic.

We could show that a part of the wavelet program can be implemented in Hardy and weighted Bergmann spaces if we consider as starting point the group generated by the composition of the Blaschke functions.

We investigated the continuous hyperbolic wavelet transforms generated by the representations of the Blaschke group on Hardy spaces, weighted Bergman spaces and their discretizations. In this way we coud construct MRA and analytic wavelets in these spaces. We could prove new atomic decomposition results in weighted Bergman spaces. Observing the relation between the HWT and Zernike polynomials we could derive the addition formula for Zernike polynomials.

1 Febr 16:25

Kahane's Upper Density and Syndetic Sets in LCA Groups

Szilárd Gy. Révész

Rényi Institute of Mathematics

Asymptotic uniform upper density, shortened as a.u.u.d., or simply upper density, is a classical notion which was first introduced by Kahane for sequences in the real line.

Syndetic sets were defined by Gottschalk and Hendlund. For a locally compact group G, a set $S \subset G$ is syndetic, if there exists a compact subset $C \Subset G$ such that $SC = G$. Syndetic sets play an important role in various fields of applications of topological groups and semigroups, ergodic theory and number theory. A lemma in the seminal book of Fürstenberg says that once a subset $A \subset \mathbb{Z}$ has positive a.u.u.d., then its difference set $A - A$ is syndetic.

The construction of a reasonable notion of a.u.u.d. in general locally compact Abelian groups (LCA groups for short) was not known for long, but in the late 2000's several constructions were worked out to generalize it from the base cases of $\mathbb{Z}^d$ and $\mathbb{R}^d$. With the notion available, several classical results of the Euclidean setting became accessible even in general LCA groups.

Here we work out various versions in a general locally compact Abelian group G of the above mentioned classical statement of Fürstenberg.

30 Jan 16:00

Sampling theorems with derivatives in shift-invariant spaces generated by exponential B-splines

Irina Shafkulovska

University of Vienna

In this talk, we discuss a new result on sufficient conditions for sampling with derivatives in shift-invariant spaces generated by an exponential B-spline. The sufficient conditions are expressed by a new notion of measuring the gap between consecutive points. As a consequence, we can construct sampling sets arbitrarily close to necessary conditions. This is joint work with K. Gröchenig.

Eigenvalue estimates for Fourier concentration operators on two domains

31 Jan
12:15

Michael Speckbacher
University of Vienna

In this talk, we consider concentration operators associated with either the discrete or the continuous multidimensional Fourier transform, that is, operators that incorporate a spatial cut-off and a subsequent frequency cut-off to the Fourier inversion formula. We derive eigenvalue estimates that quantify the extent to which Fourier concentration operators deviate from orthogonal projectors by bounding the number of eigenvalues away from 0 and 1 in terms of the geometry of the spatial and frequency domains, and a factor that grows at most poly-logarithmically on the inverse of the spectral margin. The estimates are non-asymptotic and almost match asymptotic benchmarks. Our work covers for the first time non-convex and non-symmetric spatial and frequency concentration domains, as demanded by numerous applications

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Linear independence of coherent systems associated to discrete subgroups

1 Febr
15:15am

Jordy Timo van Velthoven
University of Vienna

The Heil-Ramanathan-Topiwala (HRT) conjecture asserts that any finite family of time-frequency shifts of a nonzero square-integrable function on Euclidean space is linearly independent. One of the most fundamental contributions towards the HRT conjecture so far is due to Linnell, who showed that the conjecture is true for subsets of discrete subgroups of Euclidean space. In this talk, I will outline an elementary proof of Linnell's theorem and present an extension of this result to nilpotent Lie groups.

The talk is based on joint work with U. Enstad.
