Abstracts of the Pintz75 Workshop

Joni Teräväinen: Prime factorisations of consecutive integers

Abstract: We will discuss recent progress on several conjectures of Erdős and collaborators concerning the arithmetic function $\omega(n)$, including a conjecture of Erdős and Straus on long strings of integers with few prime factors, Erdős's irrationality conjecture for a series involving $\omega(n)$, and the Erdős–Pomerance–Sárközy conjecture on the frequency of solutions to $\omega(n) = \omega(n+1)$. A common theme is the interplay between probabilistic methods, sieves, and quantitative correlation estimates for multiplicative functions. I will outline how these tools allow us to resolve the first two conjectures and to verify the third for almost all values of x. This is based on joint work with Terence Tao.

Paul Nelson: Equidistribution and moments of L-functions

Abstract: I will survey how integral representations and period formulas recast equidistribution problems (for instance, involving unipotent shears) as moment problems for automorphic L-functions, a mechanism originating in earlier work of many authors. I will recall rank-one prototypes in which the varying vectors are pure translates of fixed ones (e.g., the triple-product setting), as well as the microlocal framework from my joint work with Venkatesh, where recent progress in homogeneous dynamics now yields effective consequences. I will then indicate a higher-rank variant, joint with Subhajit Jana, expressing $\mathrm{GL}(n)$ moments appear as L^2-norms of shears of fixed automorphic forms; here, implications for moments and subconvexity remain conditional on dynamical input not yet available.

Árpád Tóth: On a method of Hurwitz and its applications

Abstract: I will describe a method underlying a posthumous paper of Hurwitz to give class number formulas for binary cubic and n-ary quadratic forms. I will also outline how the same method can be applied to give identities for certain multiple zeta values attached to symmetric cones. The talk is based on a recent paper with Bill Duke and Ozlem Imamoglu.

Julia Stadlmann: Primes in arithmetic progressions and bounded gaps

Abstract: How many primes up to x are contained in the arithmetic progression $n \equiv a \pmod q$? What is the smallest integer that appears infinitely often as the gap between k primes? At a glance these two problems may seem somewhat unrelated. However, equidistribution estimates for primes in arithmetic progressions are a key ingredient of the GPY method of Goldston, Pintz and Yıldırım, and Zhang's breakthrough on bounded gaps between primes. In my talk I will illustrate this link between primes in APs and bounded gaps, and will discuss how new equidistribution estimates can be used to improve on the bounded gaps results of Polymath.

Henryk Iwaniec: Exponential sums for monomials in two variables with perturbations

Abstract: Sums over a box of the exponentials $e(f(m,n))$ where $f(x,y)$ is a monomial of special degrees is estimated by means of a new combination of differencing method and forging the resulting eight variables into two blocks. Then an application of the Large Sieve Inequality yields results which are stronger than these obtainable by direct application of the van der Corput method. Special formula is devised to handle unwanted perturbations.

Philippe Michel: Bilinear sums with trace functions

Abstract: We obtain non-trivial bounds for bilinear sums involving the trace function of an ℓ-adic sheaf over a finite field $\mathbb{F}_q$, which go well below the Pólya-Vinogradov range, under simple structural assumptions on its geometric monodromy group (for instance it being simple).

This is in contrast to previous works which required rather precise informations on the sheaf (such as the shape of its local monodromies at specific points along with global informations).

Our approach is "soft" and builds on the approach of moments due to J. Xu, a robust version of the Goursat-Kolchin-Ribet criterion and on Sawin's quantitative sheaf theory. In this talk we will describe this approach along with applications notably to the evaluation of cubic mixed moments of Dirichlet L-functions:

$$M_{a,b,c}(q) := \frac{1}{q-1} \sum_{\chi \bmod q} L(1/2,\chi^a)L(1/2,\chi^b)L(1/2,\chi^c)$$

This is joint work with Etienne Fouvry, Emmanuel Kowalski and Will Sawin: https://arxiv.org/abs/2511.09459

Jasson Vindas: The optimal Malliavin remainder problems for Beurling generalized numbers

Abstract: Let $\pi(x)$ and $N(x)$ denote the prime and integer counting functions of a Beurling generalized number system.

In 1961 P. Malliavin discovered that the two asymptotic relations

$$\pi(x) = \mathrm{Li}(x) + O(x \exp(-c \log^{\alpha} x)) \tag{P_α}$$

and

$$N(x) = \rho x + O(x \exp(-c' \log^{\beta} x)) \qquad (\rho > 0), \tag{N_β}$$

for some $c > 0$ and $c' > 0$, are closely related to each other in the sense that if (N_β) holds for a given $0 < \beta \leq 1$, then (P_α) is satisfied for some α, and vice versa the relation (P_α) for a given $0 < \alpha \leq 1$ ensures that (N_β) holds for a certain β. A natural question is then what the optimal remainder terms of Malliavin type are. Such a question was essentially raised by P. Bateman and H. Diamond in 1969.

In this talk we discuss what is known so far about the problem.

Jerzy Kaczorowski: Twists of L-functions — converse theorems and moments

Abstract: The talk focuses on twists of L-functions in the Selberg class and their applications to converse theorems and moment problems. Studying twists provides a powerful method for uncovering the internal structure of L-functions. The lecture will present both earlier and recent results illustrating this phenomenon, with particular emphasis on the classification of low-degree L-functions. The newer material includes an examination of self-dual twists, whose expected analytic properties could lead to sharp asymptotic formulas for moments on the critical line.

Lajos Hajdu: Minimal common factor graphs containing all graphs of order k

Abstract: A common factor graph $G = \mathcal{G}(A)$ is a finite, simple graph whose vertices are elements of a finite set $A \subset \mathbb{N}$, and two distinct vertices $a, b \in A$ are linked by an edge precisely when $\gcd(a, b) > 1$. In the talk we shall be concerned with the minimal representation of graphs as common factor graphs. After a short introduction, we shall present various results. First we show that for any graph G of order k one can find $a_1, a_2, \ldots, a_k \in \mathbb{N}$ to represent G such that the number of prime divisors of $a_1 a_2 \cdots a_k$ is at most $\lfloor k^2/4 \rfloor$, and that this value is best possible. Then we give upper and lower bounds (which are 'relatively close') for the smallest n such that every graph of order k is an induced subgraph of the common factor graph generated by the set $\{1, 2, \ldots, n\}$. Further, we answer a question of Eggleton from 1987 concerning graphs which are extremal for this type of representability to the negative, formulate a conjecture, and provide some related numerical results.

To prove our results, we need to combine several tools: combinatorial considerations, estimates for various expressions involving primes, a theorem of Erdős, Goodman and Pósa concerning coverings of graphs by cliques, and numerical methods.

The presented new results are joint with G. Batta.

Daniel Goldston: Zeros of the Riemann Zeta-Function on the Critical Line

Abstract:The Riemann Hypothesis (RH) states that all non-trivial zeros of the Riemann zeta-function lie on the vertical "critical line" with real part 1/2 in the complex plane. While the evidence that RH is true is overwhelming, there is as yet no widely accepted path towards a proof. Being more modest, one can try to prove that there are lots of zeros on the critical line. The most successful method is due to Levinson in 1974; the best result by this method obtained in 2020 is that asymptotically at least 41.7% of the zeros are on the critical line and at least 40.7% are on the critical line and also simple zeros.

In 1973 Montgomery introduced a pair correlation method to investigate the vertical distribution of zeros. In this work he introduced the Pair Correlation Conjecture, which first suggested the zeros obey a GUE random matrix model, which is now widely believed to be true, but there is no proof in sight. In joint work starting in 2023, we have been working to remove as much as possible RH from Montgomery's results on the pair correlation method. It came as a surprise when we realized that the method not only detects simple zeros but also zeros on the critical line. For example, Montgomery proved that assuming RH at least 2/3 of the zeros are simple. We prove, assuming that all the zeros are in a thin vertical region around the critical line, that also 2/3 of the zeros are on the critical line. Using a different pair correlation of Gallager and Mueller from 1978, we prove that the Pair Correlation Conjecture implies that 100% of the zeros are simple and lie on the critical line.

This is joint work with Siegfred Baluyot, Junghun Lee, Jordan Schettler, Ade Irma Suriajaya (Chacha), and Caroline L. Turnage-Butterbaugh.