Abstracts

Focused workshop
on the Brownian web, the Brownian net and their geometry

31 Mar – 4 Apr 2025

Introductory talks

- **Rongfeng Sun** (National University of Singapore)

  Title: Coupling between the Brownian web, sticky Brownian webs, and the Brownian net

  In this talk, I will explain how the Brownian net can be constructed from the Brownian web via Poisson marking, and how the Brownian web can be constructed from the Brownian net via sampling. When multiple Brownian webs are sampled from the same Brownian net, they lead to so-called sticky Brownian webs. These results play an important role in the construction of the space-time random environment underlying the Howitt–Warren stochastic flow of kernels.

- **Jan Swart** (Czech Academy of Sciences)

  Title: The Brownian net and its meshes

  Abstract: In the original paper where the Brownian net was introduced, three equivalent different constructions of a Brownian net were given: the hopping, wedge, and mesh constructions. In this talk I review these constructions with a special focus on the meshes. I will also discuss a number of questions related to these meshes that are still unresolved.

- **Jan Swart** (Czech Academy of Sciences)

  Title: Topologies on sets of paths

Research talks

- **Luiz Renato Fontes** (University of Sao Paulo)

  Title: A stronger topology for the Brownian web

  Abstract: We propose a metric space of 'coalescing pairs of paths' on which we are able to prove in a fairly direct way convergence of objects such as the 'persistence probability' in the (one dimensional, nearest neighbor, symmetric) voter model or the diffusively rescaled 'weight distribution' in a silo model (as well as the equivalent 'output distribution' in a river basin model), interpreted in terms of (dual) diffusively rescaled coalescing random walks, to corresponding objects defined in terms of the Brownian web.

- **Nic Freeman** (University of Sheffield)

  Title: Weaves, webs and flows

  Abstract: We consider "weaves" – loosely, a weave is a set of non-crossing cadlag paths that covers $1 + 1$ dimensional space-time. Here, we do not require any particular distribution for the particle motions. Weaves are a general class of random processes, of which the Brownian web is a canonical example; just as Brownian motion is a canonical example of a (single) random path. It turns out that the space of weaves has an interesting geometric structure in its own right, which will be the focus of the talk. This structure provides key information that leads to a weak convergence theory for general weaves. Joint work with Jan Swart.

- **Julian Ransford** (University of Cambridge)

  Title: The Seppäläinen–Johansson distance

  Abstract: I will discuss an intriguing connection between two topics of probability which at first appear to be unrelated: the Brownian web and the Kardar–Parisi–Zhang universality class. At the heart of this relationship is an exactly solvable model of first-passage percolation, the Seppäläinen–Johansson model. Under KPZ scaling, the model converges to the Airy process, but under Brownian scaling, it converges to a distance function on the Brownian web. This is partly based on ongoing joint work with Bálint Vető and Bálint Virág.

- **Emmanuel Schertzer** (University of Vienna)

  Title: The Brownian marble

  Abstract: We study a self-similar spatial fragmentation-coagulation process $(I_t)_{t\geq 0}$ on $\mathbb{R}$ which we call the **Brownian marble**, constructed as follows. Roughly speaking, for each $t \geq 0$, I_t is a collection $\{[a_i, b_i) : i \in J \text{ countable}\}$ of disjoint half-open intervals whose union is a subset of $\mathbb{R}$. The boundary between consecutive intervals fluctuates according to a Brownian motion, and at rate $\lambda/(b-a)^2$, an interval $[a, b)$ fragments into infinitely many small pieces. We make sense of this process starting from dust, and show that the process comes down from infinity if and only if the fragmentation parameter satisfies $\lambda < 6$. The process $(I_t)_{t\geq 0}$ has a self-similarity property in space and time, and can be treated as a random subset of the Brownian web. We give an explicit description of the space-time correlations of this process, which can be described in terms of an object we call the Brownian vein, which is a spatial version of a recurrent extension of a killed Bessel-3 process. We obtain an explicit probabilistic description for the past, present and future size of an interval containing a point x in $\mathbb{R}$ at a time t.

  This is joint work with S. Johnston, A. Kyprianou and T. Rogers.

- **Rongfeng Sun** (National University of Singapore)

  Title: Howitt–Warren flows and the ν-Brownian castle

  Abstract: I will explain ongoing work with G. Cannizzaro and M. Hairer on the nu-Brownian castle, which is a family of one-dimensional random growth models parametrized by a finite measure ν on $[0, 1]$. The special case $\nu = 0$ corresponds to the Brownian castle constructed by Cannizzaro and Hairer. For each nonzero ν, the corresponding ν-Brownian castle interpolates between the Brownian castle in the small scale limit and Edwards–Wilkinson fluctuations in the large scale limit. I will explain how the ν-Brownian castles are constructed from a family of stochastic flow of kernels called the Howitt–Warren flows, and how Howitt–Warren flows are constructed using the Brownian web and net.

- **Jan Swart** (Czech Academy of Sciences)

  Title: Augmented Brownian webs as the scaling limit of non-simple coalescing random walks

  Abstract: In my talk I will discuss coalescing non-simple random walks on the integers, started from every point in space-time, whose jump rates have finite moments up to order α. In the regime $\alpha > 3$ it is known that their scaling limit is the Brownian web. This result, which is known to be sharp, is due to Belhaouari, Mountford, Sun, and Valle (2006) and holds with respect to the standard Brownian web topology. Berestycki, Garban, and Sen (2015) showed that if one uses the weaker tube topology, that ignores the behaviour of paths near their starting times, then it suffices if $\alpha \geq 2$. I will report on work in progress with Nic Freeman where we try to understand what is going on in the intermediate regime. I will sketch a proof that in the regime $9/4 < \alpha \leq 3$, tightness with respect to the Brownian web topology is lost due to paths making a single large jump near their starting time. Tightness can be restored by allowing paths with jumps and using Skorohod's J_1 topology. In this case the scaling limit is an augmented Brownian web. In the regime $\alpha \leq 9/4$ paths appear that make two or more macroscopic jumps and it is not clear to us yet how to describe the scaling limit. Our proofs are based on a novel multi-scale decomposition of coalescing random walks.

- **Bálint Vető** (Budapest University of Technology and Economics and HUN-REN Alfréd Rényi Institute of Mathematics)

  Title: Brownian web distance and Bernoulli-Exponential first passage percolation

  Abstract: The random walk web distance is a natural directed distance on the trajectory of coalescing simple random walks. It is given by the number of jumps between different random walk paths when one is only allowed to move in one direction. The Brownian web distance is the scale-invariant limit of the random walk web distance which can be described in terms of the Brownian web. It is integer-valued and has scaling exponents 0:1:2 as compared to 1:2:3 in the KPZ world. The shear limit of the Brownian web distance is still given by the Airy process. The Bernoulli-Exponential first passage percolation is a version of random walk web distance where jumps between random walk paths are weighted by independent standard exponentials. The rescaled distance in directions close to horizantal converges to a new explicit distribution which interpolates between the Gaussian and the GUE Tracy–Widom distribution.

  Based on joint work with Bálint Virág.